

$$y = \frac{a^2x+1}{a-1} \Rightarrow y(a-1) = a^2x+1 \Rightarrow a^2x+1-y(a-1)=0 \Rightarrow a^2+(1-y)a+1-y=0$$

۲. این منحنی مرتب فوکتی!

$$\Delta = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{(y-1) \pm \sqrt{(1-y)^2 - 4(1-y)}}{2} \Rightarrow (1-y)^2 - 4(1-y) \geq 0$$

$$\rightarrow y^2 - 2y + 1 - 4 + 4y \geq 0 \rightarrow y^2 + 2y - 3 \geq 0 \rightarrow (y-1)(y+3) \geq 0$$

$\rightarrow y \in (-\infty, -3] \cup [1, +\infty) = R_f$

الف) $R_f = (\min, +\infty)$

$$\rightarrow \frac{b}{a} = \frac{5}{4} \rightarrow (1 \times \frac{5}{4}) - \frac{20}{4} + 1 = 1 - \frac{20}{4} = -\frac{14}{4}$$

$\rightarrow R_f = [-\frac{14}{4}, +\infty)$

ب) $\Delta = 9 + 18 - 5 = 22$
 $\rightarrow R_f = (-\infty, 2] \rightarrow \sqrt{(-\infty, 2]} = [2, +\infty) = R_f$

ج) $IR = \text{بر تاج در صفر} \rightarrow \text{درون یا بیرون}$

$\rightarrow \sqrt{R} = [0, +\infty) = R_f$

د) $a > 0 \rightarrow \log a = c$
 $\rightarrow \log a \in IR \rightarrow R_f = IR$

ه) $\frac{3x+1}{2x-4} = y \Rightarrow ad \neq bc \checkmark \rightarrow a = \frac{(-4y-1)}{2y-3}$
 $\rightarrow y \neq \frac{3}{2} \rightarrow R_f = IR - \{\frac{3}{2}\}$

و) $IR = R - \{\frac{a}{c}\} = R - \{\frac{1}{2}\}$
 $\rightarrow \sqrt{R - \{\frac{1}{2}\}} = [0, +\infty) - \{\frac{1}{2}\} = R_f$

ز) $y = \frac{a^2x+1}{\sqrt{a^2x+2}} = \sqrt{a^2x+2} + \frac{1}{\sqrt{a^2x+2}} \rightarrow a + \frac{1}{a} \geq 2$
 $\rightarrow \min = \sqrt{2} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \frac{\sqrt{2}}{1}$
 $\rightarrow R_f = [\frac{\sqrt{2}}{1}, +\infty)$

ح) $y + \frac{1}{x} = a^2 + \frac{1}{a^2x} \rightarrow a + \frac{1}{a} \geq 2$
 $\rightarrow y + \frac{1}{x} = \frac{1}{x} + \frac{1}{x} = \frac{2}{x} \rightarrow y = \frac{1}{x}$
 $\rightarrow y + \frac{1}{x} = \infty \rightarrow y = \infty$
 $\rightarrow R_f = [\frac{1}{x}, +\infty)$

ط) $y = \frac{3a^2x+2}{a^2-1} \rightarrow y(a^2-1) = 3a^2x+2 \rightarrow 3a^2x+2-y(a^2-1)=0 \rightarrow (3-y)a^2+y+2=0$
 $\rightarrow \Delta = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{0 \pm \sqrt{2(3-y)(y+2)}}{2(3-y)} \rightarrow 0 \geq 0$
 $\rightarrow R_f = (-\infty, -2] \cup [3, +\infty)$

ث) $\square = a^2 \rightarrow \square \in [0, +\infty) \rightarrow f(0) = -2, f(+\infty) = +\infty \rightarrow$

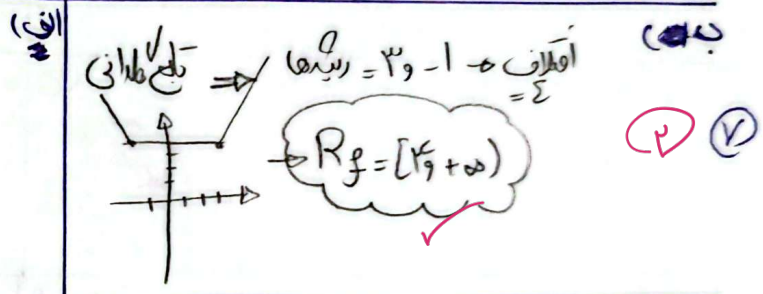
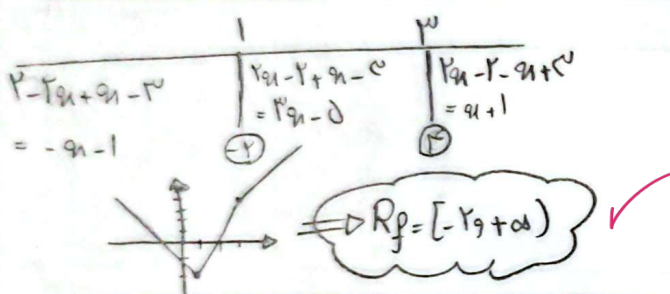
ج) $R_f = (-\infty, -2] \cup [3, +\infty)$

Q1: $y = \frac{15 \sin \alpha - 1}{\sin \alpha + 1} \rightarrow y \sin \alpha + y = 15 \sin \alpha - 1 \rightarrow 15 \sin \alpha - y \sin \alpha - y - 1 = 0 \rightarrow (15 - y) \sin \alpha = y + 1$
 $\rightarrow \frac{y+1}{15-y} = \sin \alpha \rightarrow \frac{y+1}{15-y} \geq -1 \rightarrow \frac{y+1}{15-y} \leq 1 \rightarrow \frac{y+1}{15-y} \geq 0$
 $\rightarrow \frac{y+1}{15-y} \geq 0 \rightarrow \frac{y+1}{15-y} \leq 0 \rightarrow \frac{y+1}{15-y} \geq 0$

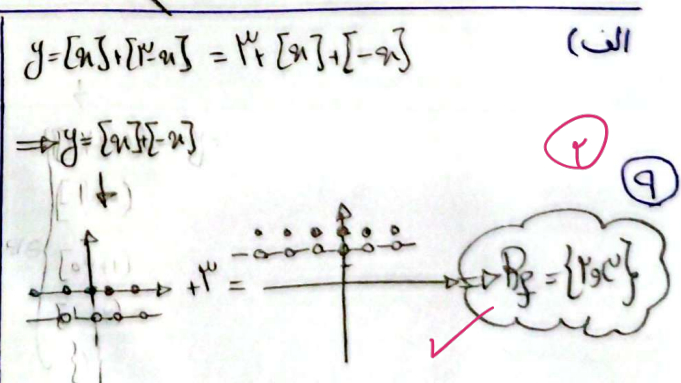
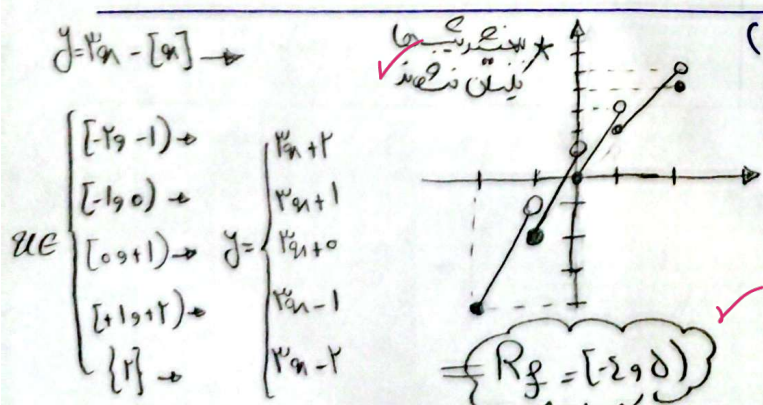
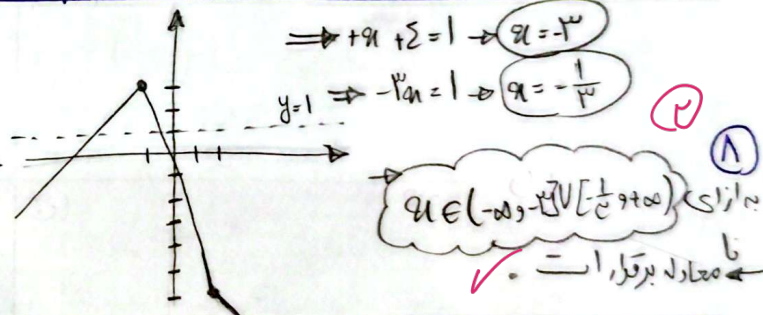
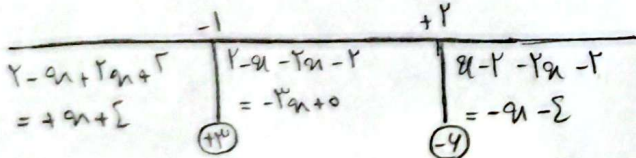
Q2: $y = \sin \alpha \rightarrow y \in [-1, 1] \rightarrow f(-1) = \frac{1}{2}, f(1) = \frac{1}{2} \Rightarrow R_f = [\frac{1}{2}, \frac{1}{2}]$

$\square = a^2 - 9a \rightarrow \frac{-b}{2a} = \frac{1}{2} \rightarrow \frac{1}{2} - \frac{1}{2} = -\frac{1}{2} \rightarrow R_g = [-\frac{1}{2}, +\infty) \rightarrow \square \in [-\frac{1}{2}, +\infty)$

$\rightarrow f(x) = \frac{x + \frac{1}{2}}{x + 1} \rightarrow f(\frac{1}{2}) = \frac{0}{\frac{3}{2}} = 0, f(+\infty) = 1 \rightarrow x \neq -1$
 $\rightarrow R_f = [0, 1)$



$|x - 1| - |2x + 1| = y$



$y = \sin^2 \alpha + \sin \alpha + 1$
 $\min \rightarrow 0 \rightarrow y = 1$
 $\max \rightarrow 1 \rightarrow y = 3$
 $\rightarrow R_f = [1, 3]$

$y = \sin^2 \alpha - \sin \alpha + 1$
 $\min \rightarrow -1 \rightarrow y = 3$
 $\max \rightarrow 1 \rightarrow y = 1$
 $\rightarrow R_f = [1, 3]$