نام و نام خانوادگی .... امیر محمد ابوطالبی .... پاسخنامه تشریحی تکلیف شماره .... کلاس ....

۱۷/۷۵

۱ (۲)

$y = \frac{x^2+x+2}{x-1} \longrightarrow yx - y = x^2+x+2$

$\therefore x^2+x-yx = -2-y \rightarrow x^2 + (1-y)x + (y+2) = 0$

$x = \frac{y-1 \pm \sqrt{y^2-6y-7}}{2} \therefore$ چون جواب معادله باید ریشه داشته باشد $\longrightarrow y^2-6y-7 \geq 0 \rightarrow (y-7)(y+1) \geq 0$

$\therefore \quad + \; -1 \; - \; 7 \; + \longrightarrow R_y = (-\infty,-1] \cup [7,+\infty)$

۲ (۱/۲۵)

الف) $y = 2x^2-5x+1 \longrightarrow 2y = 4x^2-10x+2 \longrightarrow 2y = (2x-2.5)^2 - 4.25$

$2y+4.25 = (2x-2.5)^2 \longrightarrow \sqrt{2y+4.25} = 2x-2.5 \longrightarrow \sqrt{2y+4.25}+2.5 = 2x \rightarrow x = \frac{\sqrt{2y+4.25}+2.5}{2}$

$2y+4.25 \geq 0 \longrightarrow 2y \geq -4.25 \rightarrow \boxed{y \geq -2.125}$

ب) $y = \sqrt{-x^2+6x-5} \longrightarrow -y^2 = x^2-6x+5 \rightarrow -y^2+4 = (x-3)^2 \rightarrow \sqrt{-(y^2-4)} = x-3$

$x = \sqrt{-(y^2-4)}+3 \longrightarrow -(y^2-4) \geq 0 \longrightarrow 4-y^2 \geq 0 \rightarrow (2-y)(2+y) \geq 0 \rightarrow \; - \; -2 \; + \; 2 \; -$

عبارت زیر رادیکال $R = [2,-2] \rightarrow R = [0,2]$ $\quad R_f = [0,+\infty)$

ج) $y = \sqrt{x^4-4x^3+4x+1} \longrightarrow$ برد R $\quad \log(x^4-4x^3+7x+1) \quad \log_1(R) \rightarrow R_f = (-\infty,+\infty)$

۳ (۱/۲۵)

الف: $y = \frac{3x+1}{2x-6} \Rightarrow x = -\frac{-6y-1}{2y-3} \rightarrow x = \frac{6y+1}{2y-3} \rightarrow 2y-3 \neq 0 \rightarrow y \neq \frac{3}{2} \rightarrow R-\{\frac{3}{2}\}$

$y = [0,+\infty) - \{2\}$

ب) $y = \sqrt{\frac{4x+1}{x-2}} \rightarrow y^2 = \frac{4x+1}{x-2} \rightarrow x = -\frac{-2y^2-1}{y^2-4} \rightarrow x = \frac{2y^2+1}{y^2-4} \longrightarrow y^2-4 \neq 0 \rightarrow y \rightarrow R-\{2,-2\}$

چون رادیکال همواره مثبت است یا صفر است.

$f\,min = \sqrt{3} + \frac{1}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$

ج) $y = \frac{x^2+4}{\sqrt{x^2+3}} = \frac{x^2+3}{\sqrt{x^2+3}} + \frac{1}{\sqrt{x^2+3}} = \sqrt{x^2+3} + \frac{1}{\sqrt{x^2+3}} \xrightarrow{t+\frac{1}{t} \geq 2} R_y = [2,+\infty)$

$R_f = [\frac{4\sqrt{3}}{3}, +\infty)$

$y = x^2 + \frac{1}{x^2+4} \rightarrow x^2 + \frac{1}{x^2+4} + 4 - 4 \longrightarrow \left(x^2+4+\frac{1}{x^2+4}\right) - 4 \longrightarrow R_y = [-2,+\infty)$

$f\,min = \frac{1}{4} \quad R_f = [\frac{1}{4},+\infty)$

۴ (۲)

$y = \frac{3x^2+4}{x^2-1} \rightarrow yx^2 - y = 3x^2+4 \rightarrow 3x^2 - yx^2 = -y-4 \rightarrow x^2 = \frac{-y-4}{3-y}$

$x = \sqrt{\frac{y+4}{y-3}} \longrightarrow \frac{y+4}{y-3} \geq 0 \rightarrow$ ~~...~~ $, \; y-3 \neq 0 \rightarrow y \neq 3$

$+ \; -4 \; - \; 3 \; +$

~~$R_y = [\ldots]$~~ $R_f = (-\infty,-4] \cup (3,+\infty)$

$y = \frac{3x^2+4}{x^2-1} \xrightarrow{x^2=0} \frac{4}{-1} = -4 \;,\; \frac{3x^2+4}{x^2-1} \xrightarrow{x^2=+\infty} \frac{3\infty}{\infty} = 3 \rightarrow$ خروجی نمی‌تواند صفر شود $\rightarrow (-\infty,-4] \cup (3,+\infty)$

۵ (۲)

$y = \frac{2\sin x - 1}{\sin x + 3} \rightarrow \sin x = -\frac{3y+1}{y-2} \rightarrow \sin x = \frac{3y+1}{2-y}$

$-1 < \frac{3y+1}{2-y} < 1 \longrightarrow \frac{3y+1}{2-y} < 1 \rightarrow 3y+1 < 2-y \rightarrow 4y < 1 \rightarrow y < \frac{1}{4}$

$\rightarrow -1 < \frac{3y+1}{2-y} \rightarrow y-2 < 3y+1 \rightarrow -3 < 2y \rightarrow -\frac{3}{2} < y$

$R_y = -\frac{3}{2} < y < \frac{1}{4}$

$y = \frac{2\sin x-1}{\sin x+3} \xrightarrow{\sin x=1} \frac{1}{4} \;,\; \frac{2\sin x-1}{\sin x+3} \xrightarrow{\sin x=-1} \frac{-3}{2} \rightarrow R_y = -\frac{3}{2} < y < \frac{1}{4}$

خروجی صفر نمی‌شود

$Rf = [0,1)$

$y = f(x) = \frac{x^2 - x + \frac{1}{4}}{x^2 - x + 1}$ $\qquad$ ~~$f(x) = \frac{(x^2-x)+\frac14}{(x^2-x)+1}$~~ $\qquad$ مفرجم $y \neq 1$

~~$f(x) \xrightarrow{x \to +\infty} \frac{\infty}{\infty} = 1$~~ $\qquad x = \frac{(y-1) \pm \sqrt{(y-1)^2 - 4(y-\frac14)(y-1)}}{2(y-1)} \to 0 \le y < 1$

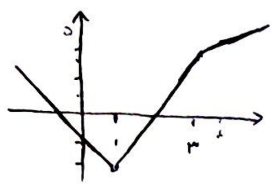
$f(x) \to y = \frac{x^2 - x + \frac14}{x^2 - x + 1} \to yx^2 - yx + y = x^2 - x + \frac14 \to (1-y)x^2 + (y-1)x + \frac14 - y = 0$ $\quad$ ۱/۵ $\quad$ ۶

$R_f = [\frac14, 1]$ $\quad \to x = \frac{1-y \pm \sqrt{3y - 3y^2}}{2-2y} \to 3y - 3y^2 \ge 0 \to 3y(1-y) \ge 0$

لازم نیست برای

$-\ \tfrac13\ +\ 0\ -$

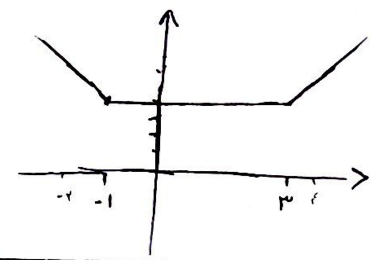
الف) $y = |2x-2| - |x-3|$ $\qquad$ $-x-1 \;\big|_1\; 3x-5 \;\big|_3\; x+1$



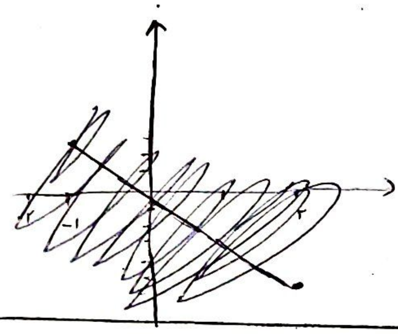
$\to R_y = [-2, +\infty)$ $\quad$ ۲

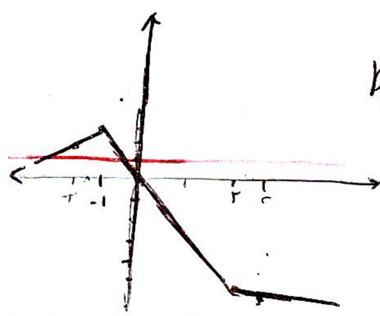
ب) $y = |x-3| + |x+1|$ $\to$ $\qquad$ $-2x+2 \;\big|_{-1}\; 4 \;\big|_3\; 2x-2 \to$ $\quad$ ۷

$R_y = [4, +\infty)$



$|x-2| - |2x+2| \le 1 \to |x-2| - |2x+2| \to$ $\qquad$ $x+4 \;\big|_{-1}\; -3x \;\big|_2\; -x-4$





$D_f = R - (-3, -\frac13)$ $\quad$ ۲ $\quad$ ۸

الف) $y = [x] + [3-x]$ $\quad Rf = \{3, 2\}$

$x = 2 \to y = 3$

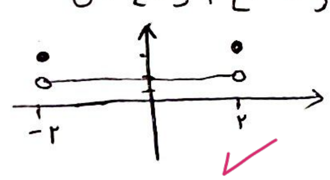
$1 \le x < 2 \to y = 1 + 1 = 2$

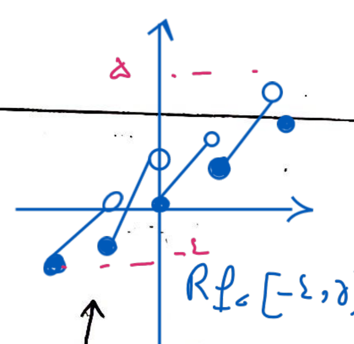
$0 < x \le 1 \to y = 0 + 2 = 2$

$-1 \le x < 0 \to y = -1 + 3 = 2$

$-2 < x \le -1 \to y = -2 + 4 = 2$

$x = -2 \to y = -2 + 5 = 3$





$Rf = [-4, 5)$

ب) $y = 3x - [x]$ $\quad$ ۱/۲۵ $\quad$ ۹

$x = 2 \to 3x - 2 = 4$

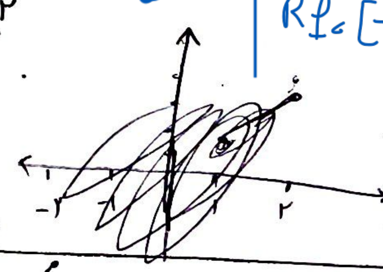
$1 \le x < 2 \to 3x - 1$

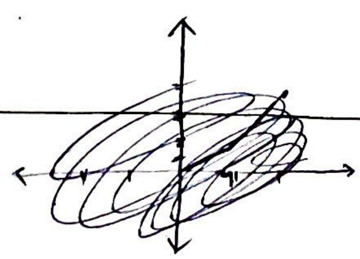
$0 \le x < 1 \to 3x$

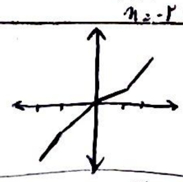
$-1 \le x < 0 \to 3x + 1$

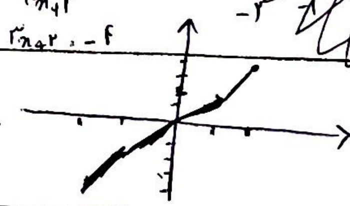
$-2 < x < -1 \to 3x + 2$

$x = -2 \to 3x + 2 = -4$









۶

الف) $y = \sin^2 x - \sin x + 1 \longrightarrow y \begin{cases} \sin x = 1 \to y = 1 \\ \sin x = -1 \to y = 3 \\ \frac{-b}{2a} = \frac12 \to y = \frac34 \end{cases} \to y = [\frac34, 3]$ $\quad$ ۱۰

ب) $y = \sin^4 x + \sin^2 x + 1 \xrightarrow{\sin^2 x = \square} y \begin{cases} \sin^2 x = 1 \to y = 3 \\ \sin^2 x = 0 \to y = 1 \\ \square = -\frac12 \to y = \frac{11}{8} \end{cases} \to y = [1, 3]$ $\quad$ ۲