نام و نام خانوادگی .... امیرمحمد ابوطالبی **پاسخنامه تشریحی تکلیف شماره .... کلاس ....**

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$y = \frac{x^2+x+2}{x-1} \longrightarrow yx - y = x^2 + x + 2$

$\therefore x^2 + x - yx = -2 - y \to x^2 + (1-y)x + (y+2) = 0$

$x = \frac{y-1 \pm \sqrt{y^2-6y-7}}{2} \therefore \xrightarrow[\text{داشته باشد}]{\text{چون جواب معادله باید}} y^2 - 6y - 7 \geq 0 \to (y-7)(y+1) \geq 0$

$\therefore$ $+ \; -1 \; - \; 7 \; +$ $\longrightarrow R_y = (-\infty, -1] \cup [7, +\infty)$

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الف) $y = 2x^2 - 5x + 1 \longrightarrow 2y = 4x^2 - 10x + 2 \longrightarrow 2y = (2x - 2.5)^2 - 4.25$

$2y + 4.25 = (2x - 2.5)^2 \longrightarrow \sqrt{2y+4.25} = 2x - 2.5 \longrightarrow \sqrt{2y+4.25} + 2.5 = 2x \to x = \frac{\sqrt{2y+4.25}+2.5}{2}$

$2y + 4.25 \geq 0 \longrightarrow 2y \geq -4.25 \to \boxed{y \geq -2.125}$

ب) $y = \sqrt{-x^2+6x-5} \to -y^2 = x^2 - 6x + 5 \to -y^2 + 4 = (x-3)^2 \to \sqrt{-(y^2-4)} = x - 3$

$x = \sqrt{-(y^2-4)} + 3 \longrightarrow -(y^2-4) \geq 0 \longrightarrow 4 - y^2 \geq 0 \to (2-y)(2+y) \geq 0 \to$ $- \; -2 \; + \; 2 \; -$

$\boxed{R = [2, -2]}$

پ) $y = \underbrace{\sqrt{x^4 - 4x^3 + 4x + 1}}_{\text{برد } R} \longrightarrow R_f = [0, +\infty)$ ت) $\log(x^4 - 4x^3 + 7x + 1)$ $\xrightarrow{\text{برد } R}$

$\log_1 (R) \to R_f = (-\infty, +\infty)$

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الف: $y = \frac{3x+1}{2x-6} \Rightarrow x = -\frac{-6y-1}{2y-3} \to x = \frac{6y+1}{2y-3} \to 2y - 3 \neq 0 \to y \neq \frac{3}{2} \to R - \{\frac{3}{2}\}$

ب: $y = \sqrt{\frac{4x+1}{x-2}} \to y^2 = \frac{4x+1}{x-2} \to x = -\frac{-2y^2-1}{y^2-4} \to x = \frac{2y^2+1}{y^2-4} \to y^2 - 4 \neq 0 \to y \to R - \{2, -2\}$

پ: $y = \frac{x^2+4}{\sqrt{x^2+3}} = \frac{x^2+3}{\sqrt{x^2+3}} + \frac{1}{\sqrt{x^2+3}} = \overbrace{\sqrt{x^2+3}}^{t} + \frac{1}{\underbrace{\sqrt{x^2+3}}_{t}} \xrightarrow{t + \frac{1}{t} \geq 2} R_y = [2, +\infty)$ $y \geq 2$

$y = x^2 + \frac{1}{x^2+4} \to x^2 + \frac{1}{x^2+4} + 4 - 4 \to (x^2 + 4 + \frac{1}{x^2+4}) - 4 \longrightarrow R_y = [-2, +\infty)$

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$y = \frac{3x^2+4}{x^2-1} \to yx^2 - y = 3x^2 + 4 \to 3x^2 - yx^2 = -y - 4 \to x^2 = \frac{-y-4}{3-y}$

$x = \sqrt{\frac{y+4}{y-3}} \longrightarrow \frac{y+4}{y-3} \geq 0 \to$ ~~[illegible]~~ , $y - 3 \neq 0 \to y \neq 3$

$+ \; -4 \; - \; 3 \; +$

~~$R_y = [\ldots]$~~ $R_f = (-\infty, -4] \cup (3, +\infty)$

$y = \frac{3x^2+4}{x^2-1} \xrightarrow{x^2 = 0} \frac{4}{-1} = -4$, $\frac{3x^2+4}{x^2-1} \xrightarrow{x^2 \to +\infty} \frac{3\infty}{\infty} = 3 \to$ خروجی می‌تواند مرز نشود $\to (-\infty, -4] \cup (3, +\infty)$

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$y = \frac{2\sin x - 1}{\sin x + 3} \to$ ~~$\sin x$~~ $\sin x = -\frac{3y+1}{y-2} \to \sin x = \frac{3y+1}{2-y}$

$-1 < \frac{3y+1}{2-y} < 1 \longrightarrow \frac{3y+1}{2-y} < 1 \to 3y + 1 < 2 - y \to 4y < 1 \to y < \frac{1}{4}$

$\to -1 < \frac{3y+1}{2-y} \to y - 2 < 3y + 1 \to -3 < 2y \to -\frac{3}{2} < y$

$R_y = -\frac{3}{2} < y < \frac{1}{4}$

$y = \frac{2\sin x - 1}{\sin x + 3} \xrightarrow{\sin x = 1} \frac{1}{4}$, $\frac{2\sin x - 1}{\sin x + 3} \xrightarrow{\sin x = -1} \frac{-3}{2} \to R_y = -\frac{3}{2} < y < \frac{1}{4}$

خروجی صفر نمی‌شود

$y \quad f(x) = \dfrac{x^2 - x + \frac{1}{4}}{x^2 - x + 1} \qquad f(x) = \dfrac{\cancel{(x^2-x)} + \frac{1}{4}}{1(x^2-x)+1}$

~~$f(x) \xrightarrow{x \to +\infty} \frac{\infty}{\infty} = 1$~~ ۶

$f(x) \to y = \dfrac{x^2 - x + \frac{1}{4}}{x^2 - x + 1} \to yx^2 - yx + y = x^2 - x + \frac{1}{4} \to (1-y)x^2 + (y-1)x + \frac{1}{4} - y = 0$

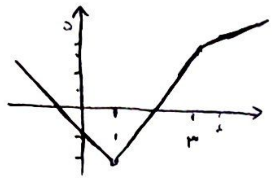
$\to x = \dfrac{1 - y \pm \sqrt{3y - 3y^2}}{2 - 2y}$ ← لازم نیست بررسی شود $\to 3y - 3y^2 \geq 0 \to 3y(1-y) \geq 0$

$- \quad 0 \quad + \quad 1 \quad -$

$R_f = [\frac{1}{4}, 1]$

الف. $y = |2x-2| - |x-3|$

$-x-1$	1	$3x-5$	3	$x+1$

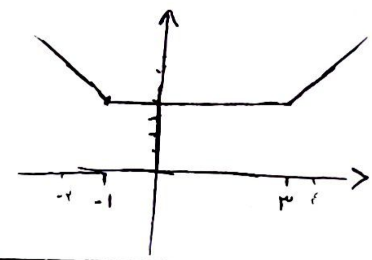


$\to R_y = [-2, +\infty)$ ۷

ب - $y = |x-3| + |x+1| \to$

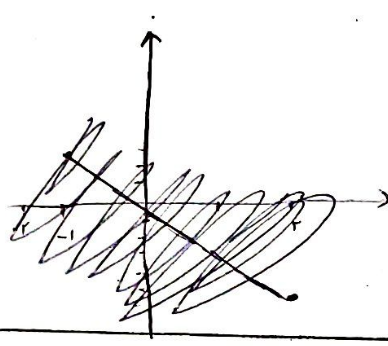
$2-2x$	-1	4	3	$2x-2$

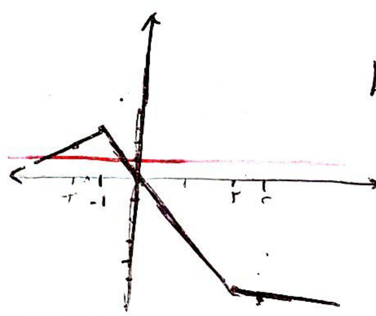
$R_y = [4, +\infty)$



$|x-2| - |2x+2| \leq 1 \to |x-2| - |2x+2| \to$

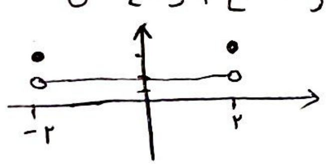
$x+4$	-1	$-3x$	2	$-x-4$





$D_f = R - (-3, -\frac{1}{3})$ ۸

الف $y = [x] + [3-x]$



$x = 2 \to y = 3$

$1 \leq x < 2 \to y = 1 + 1 = 2$

$0 \leq x < 1 \to y = 0 + 2 = 2$

$-1 \leq x < 0 \to y = -1 + 3 = 2$

$-2 < x \leq -1 \to y = -2 + 4 = 2$

$x = -2 \to y = -2 + 5 = 3$ ۹

ب $y = 3x - [x]$

$x = 2 \to 3x - 2 = 4$

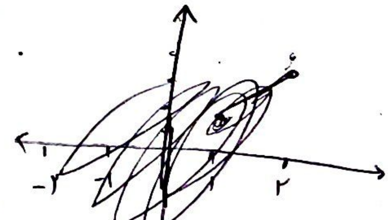
$1 \leq x < 2 \to 3x - 1$

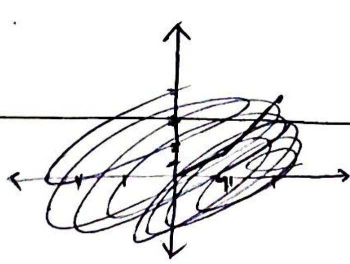
$0 \leq x < 1 \to 3x$

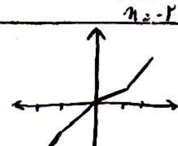
$-1 \leq x < 0 \to 3x + 1$

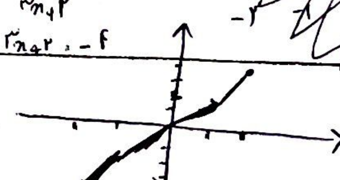
$-2 < x < -1 \to 3x + 2$

$x = -2 \to 3x + 2 = -4$









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الف $y = \sin^2 x - \sin x + 1 \longrightarrow y \begin{cases} \sin x = 1 \to y = 1 \\ \sin x = -1 \to y = 3 \\ \frac{-b}{2a} = \frac{1}{2} \to y = \frac{3}{4} \end{cases} \to y = [\frac{3}{4}, 3]$ ۱۰

ب $y = \sin^4 x + \sin^2 x + 1 \xrightarrow{\sin^2 x = \square} y \begin{cases} \sin^2 x = 1 \to y = 3 \\ \sin^2 x = 0 \to y = 1 \\ -\frac{1}{2} \to y = \frac{11}{8} \end{cases} \to y = [1, 3]$

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