

الف) $y = (2(\frac{x}{4}) - 2[\frac{x}{4}])^2 \rightarrow \frac{x}{4} = a \rightarrow (2(a - [\frac{a}{4}]))^2 \rightarrow R_y \in [0, 4]$ ✓

ب) $\sin x + 4 = y \rightarrow y \in [4, 5] \rightarrow R_y \in [1.3, 1.5]$ ✓

$f(x) = \sqrt{\frac{x^2-4}{x^2-9}} \xrightarrow{x^2=t} \sqrt{\frac{t-4}{t-9}} \rightarrow$ حتماً $R = \{ \frac{4}{9} \} \rightarrow R = \{ 1 \}$

$R_f \in [0, 1) \cup (1, +\infty)$ ← زیر رادیکال

$f(\infty) = 1 \rightarrow R_f = (-\infty, \frac{4}{9}] \cup (1, +\infty) \xrightarrow{a+b+c, \frac{a}{b}}$
 $f(0) = \frac{4}{9}$

الف) $f(x) = x + \frac{1}{x} + 2\sqrt{x + \frac{1}{x}} \rightarrow x + \frac{1}{x} \in (-\infty, -1] \cup [2, +\infty) \} R_f \in [2\sqrt{2}, +\infty)$ ✓
 $x > 0 \in [2, +\infty) \quad [\sqrt{2}, +\infty)$

ب) $y = -\sin^2 x + 2\sin 4x \xrightarrow{\frac{1}{4}} R_y \in [0, 4]$ ✓

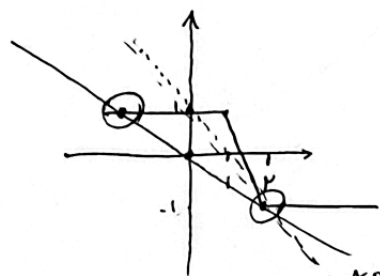
$f(x) = -\frac{1}{x}x + 1, D_f = \{x | x \in \mathbb{N}, x \leq 10\}$ ✓

$\frac{(\frac{29}{x} + \frac{1}{x}) \times 9}{x} = 10$ ✓

$1 \rightarrow \frac{29}{x}$
 $2 \rightarrow \frac{29}{x}$
 $3 \rightarrow \frac{29}{x} = 9$
 $9 \rightarrow \frac{29}{x} = 7$

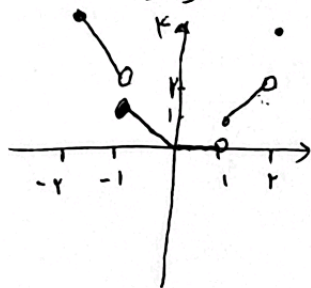
$f(x) = \sqrt{ax^2 + bx + c} = \sqrt{x-2} \rightarrow a=0, b=1, c=-2$
 $\hookrightarrow \frac{1}{4}$
 $\hookrightarrow \sqrt{4-2} = \sqrt{2} = 1$

$\hookrightarrow g(x) = \sqrt{x^2 + 4} \rightarrow R_g \in [2, +\infty)$ ✓



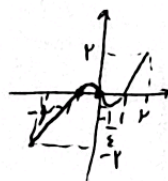
از سبب این که $kx \rightarrow y \rightarrow -1 \rightarrow kx = -\frac{1}{x}$
 $\boxed{k = -\frac{1}{x}}$

الف) $x[x]$



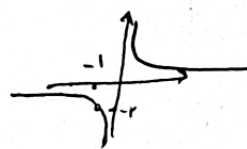
$[-2, -1) \rightarrow -2x$
 $[-1, 0) \rightarrow -x$
 $[0, 1) \rightarrow 0$
 $[1, 2) \rightarrow x$
 $\{2\} \rightarrow x$
 $R \in [-2, 2] - \{1\}$

ب) $x|x|-x \rightarrow -\frac{1}{(x^2+1)} x^2 - x$



$R \in [-2, 2]$

$$y = \frac{1}{x} + \frac{1}{x+1} + \frac{1}{x^2+x} = \frac{x+x+1+1}{x^2+x} = \frac{2(x+1)}{x(x+1)} = 2\left(\frac{1}{x}\right)$$

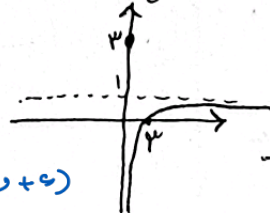


$R \in \mathbb{R} - \{-2, 0\} \rightarrow -2 + 0 = -2$

$$f(x) = \frac{(\sqrt{x}-1)^2-1}{(\sqrt{x}-1)^2} \xrightarrow{\sqrt{x}-1=t} \frac{(t-1)^2-1}{t^2} = \frac{t^2-2t+1-1}{t^2} = \frac{t-2}{t}$$

$\frac{\sqrt{x}-2}{\sqrt{x}}$

$x > 0$ $\sqrt{x} = a \rightarrow \frac{a-2}{a} \rightarrow$



$f(\infty) = 1$
 $f(0) = \infty$
 $\rightarrow$ مخرج صفر شود $R_f = (-\infty, 1) \cup [2, +\infty)$

$$f(x) = \frac{(x-1)(x+1)(x+2)}{(x-1)(x+1)} = x+2 \rightarrow x \neq 1, -1$$

$\left\{ \begin{matrix} [-1, 1] \\ 1, 2 \end{matrix} \right\} \rightarrow 1+2 = 3$