

الف) $y = (2(\frac{x}{4}) - 2[\frac{x}{4}])^2 \rightarrow \frac{x}{4} = a \rightarrow (2(a - [\frac{a}{4}]))^2 \rightarrow R_y \in [0, 4)$
 ب) $\sin x + 4 = y \rightarrow y \in [4, 5] \rightarrow R_y \in [1.3, 1.5]$

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$f(x) = \sqrt{\frac{x^2-4}{x^2-9}} \xrightarrow{x^2=t} \sqrt{\frac{t-4}{t-9}} \rightarrow \text{معرّنات} \rightarrow R - \{\frac{9}{4}\} \rightarrow R - \{1\}$
 $R_f \in [0, 1) \cup (1, +\infty)$ ← زیر رادیکال

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الف) $f(x) = x + \frac{1}{x} + 2\sqrt{x + \frac{1}{x}} \rightarrow x + \frac{1}{x} \in (-\infty, -1] \cup [2, +\infty) \} R_f \in [2\sqrt{2}, +\infty)$
 $x > 0 \in [2, +\infty) \quad [2\sqrt{2}, +\infty)$
 ب) $y = -\sin^2 x + 2\sin 4x \in [-1, 1] \} R_y \in [0, 4]$

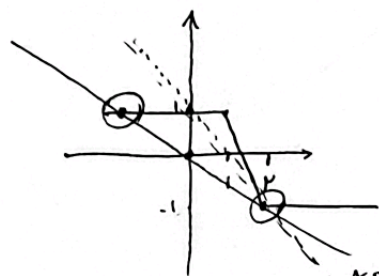
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$f(x) = -\frac{1}{x}x + 1, D_f = \{x | x \in \mathbb{N}, x \leq 10\}$
 $(\frac{29}{x} + \frac{1}{x}) \times 9 = 10$
 $1 \rightarrow \frac{29}{3}$
 $2 \rightarrow \frac{29}{4}$
 $3 \rightarrow \frac{29}{5} = 9$
 $9 \rightarrow \frac{29}{10} = 7$

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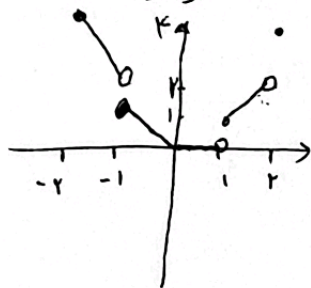
$f(x) = \sqrt{ax^2 + bx + c} = \sqrt{x-2} \rightarrow a=0, b=1, c=-2$
 $\hookrightarrow \frac{1}{4}$
 $\hookrightarrow \sqrt{4-2} = \sqrt{2} = 1$
 $\hookrightarrow g(x) = \sqrt{x^2 + 4} \rightarrow R_g \in [2, +\infty)$

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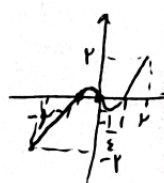
از سبب این که $kx \rightarrow y \rightarrow -1 \rightarrow kx = -\frac{1}{r}x$
 $\hookrightarrow \boxed{k = -\frac{1}{r}}$

الف) $x[x]$



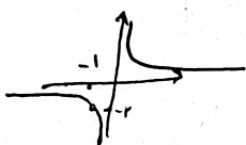
$[-2, -1) \rightarrow -2x$
 $[-1, 0) \rightarrow -x$
 $[0, 1) \rightarrow 0$
 $[1, 2) \rightarrow x$
 $\{2\} \rightarrow x$
 $R \in [0, 2] - \{2\}$

ب) $x|x| - x \rightarrow \frac{-x^2}{x^2 - x}$



$R \in [-2, 2]$

$$y = \frac{1}{x} + \frac{1}{x+1} + \frac{1}{x^2+x} = \frac{x+x+1+1}{x^2+x} = \frac{2(x+1)}{x(x+1)} = 2\left(\frac{1}{x}\right)$$

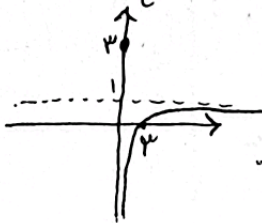


$R \in \mathbb{R} - \{-2, 0\} \rightarrow -2 + 0 = \textcircled{-2}$

$$f(x) = \frac{(\sqrt{x}-1)^2-1}{(\sqrt{x}-1)^2} \xrightarrow{\sqrt{x}-1=t} \frac{(t-1)^2-1}{t^2} = \frac{t^2-2t+1-1}{t^2} = \frac{t-2}{t}$$

$\xrightarrow{x=0} f(0)=2$

$\frac{\sqrt{x}-2}{\sqrt{x}} \xrightarrow{x>0} \frac{\sqrt{x}-2}{\sqrt{x}} \xrightarrow{\sqrt{x}=u} \frac{u-2}{u} \rightarrow$



$\rightarrow R_f \in (-\infty, 1) \cup \{2\}$

$$f(x) = \frac{(x-1)(x+1)(x+2)}{(x-1)(x+1)} = x+2 \rightarrow x \neq 1, -1$$

$\left\{ \begin{array}{l} [-1, 1] \\ [1, 2] \end{array} \right\} \rightarrow 1+2=3$