

برای حل

تکلیف می ده

« کیوان کاغذی »

ا) $y = \left(\sqrt{\frac{x}{x^2-4}} + 1 \right)^2$ $\frac{\sqrt{\frac{x}{x^2-4}} - \left[\frac{x}{x^2-4} \right] < 1}{\text{می دانیم}} \rightarrow 0 < \sqrt{\frac{x}{x^2-4}} - \left[\frac{x}{x^2-4} \right] < 1$ (1)

$\frac{2}{\text{مقدار}} \rightarrow 0 < \left(\sqrt{\frac{x}{x^2-4}} + 1 \right)^2 < 4 \rightarrow R_f = [0, 4) \checkmark$

ب) $\log y = x + \sin x$ (1) $-1 < \sin x < 1$ (2) $\rightarrow -1 < \log y + x < 1 + x$

(3) $\rightarrow -1 < \log y < 1 \rightarrow 10^{-1} < y < 10^1 \rightarrow R_f = [10^{-1}, 10^1] \checkmark$

$y = \sqrt{\frac{x^2-4}{x^2-9}}$ (1) $y^2 = \frac{x^2-4}{x^2-9} \rightarrow y^2(x^2-9) = x^2-4$ (2)

$(y^2-1)x^2 + 4 - 9y^2 = 0 \rightarrow x = \frac{\pm \sqrt{-4(y^2-1)(4-9y^2)}}{2(y^2-1)}$

$x(9y^4 - 9y^2 - 4y^2 + 4) \geq 0 \rightarrow x(9y^4 - 13y^2 + 4) \geq 0$ همه ضرایب مثبت است

پس برای $y = \frac{2}{3}$ و $y = 1$ جواب می شود

(1) $y > 0 \rightarrow y = \pm \sqrt{\frac{4}{9}} \rightarrow \pm \frac{2}{3}$ + 1 - 1 +

$\rightarrow R_f = [0, \frac{2}{3}] \cup [1, +\infty)$ $\rightarrow a=0, b=\frac{2}{3}, c=1$

$a+b+c = 0 + \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$ ✓

ا) $y = \left(\sqrt{x + \frac{1}{x}} + 1 \right)^2$ $D_f = (0, +\infty)$ $\rightarrow x + \frac{1}{x} \geq 2 \rightarrow \sqrt{x + \frac{1}{x}} \geq \sqrt{2}$

$\rightarrow \sqrt{x + \frac{1}{x}} + 1 \geq \sqrt{2} + 1$ $\frac{2}{\text{مقدار}} \rightarrow \left(\sqrt{x + \frac{1}{x}} + 1 \right)^2 \geq 3 + 2\sqrt{2} \rightarrow f(x) \geq 3 + 2\sqrt{2}$

$\rightarrow R_f = [3 + 2\sqrt{2}, +\infty) \checkmark$

ب) $y = -(\sin x - 2)(\sin x + 1) = -(\sin^2 x - 2\sin x - 2) \rightarrow y = -\sin^2 x + 2\sin x + 2$

$\Rightarrow$ (1) $\sin x = 1 \rightarrow y = -1 + 2 + 2 = 3$ (2) $\sin x = -1 \rightarrow y = -1 - 2 + 2 = -1$

(3) $\rightarrow \sin x = \frac{2}{3} \rightarrow y = -\frac{4}{9} + 2 + 2 = \frac{28}{9}$ (1, 2, 3) $R_f = [0, 4] \checkmark$

$x=1 \rightarrow y = \frac{19}{x}$ / $x=2 \rightarrow y = \frac{18}{x}$ / $x=3 \rightarrow y = \frac{17}{x}$ / $x=4 \rightarrow y = \frac{16}{x}$ / $x=5 \rightarrow y = \frac{15}{x}$ / $x=6 \rightarrow y = \frac{14}{x}$ / $x=7 \rightarrow y = \frac{13}{x}$ / $x=8 \rightarrow y = \frac{12}{x}$ / $x=9 \rightarrow y = \frac{11}{x}$ / $x=10 \rightarrow y = \frac{10}{x}$ / $x=11 \rightarrow y = \frac{9}{x}$ / $x=12 \rightarrow y = \frac{8}{x}$ / $x=13 \rightarrow y = \frac{7}{x}$ / $x=14 \rightarrow y = \frac{6}{x}$ / $x=15 \rightarrow y = \frac{5}{x}$ / $x=16 \rightarrow y = \frac{4}{x}$ / $x=17 \rightarrow y = \frac{3}{x}$ / $x=18 \rightarrow y = \frac{2}{x}$ / $x=19 \rightarrow y = \frac{1}{x}$

$\Rightarrow R_f = \left\{ \frac{19}{1}, \frac{18}{2}, 1, \frac{17}{3}, \frac{16}{4}, 1, \frac{15}{5}, \frac{14}{6}, 1, \frac{13}{7}, \frac{12}{8}, 1, \frac{11}{9}, \frac{10}{10}, 1, \frac{9}{11}, \frac{8}{12}, 1, \frac{7}{13}, \frac{6}{14}, 1, \frac{5}{15}, \frac{4}{16}, 1, \frac{3}{17}, \frac{2}{18}, 1, \frac{1}{19} \right\}$

$\omega_b = [r, +\infty) \rightarrow a=0 \rightarrow b \cdot x + c > 0 \rightarrow x > -\frac{c}{b} \rightarrow -\frac{c}{b} = r \quad (1)$

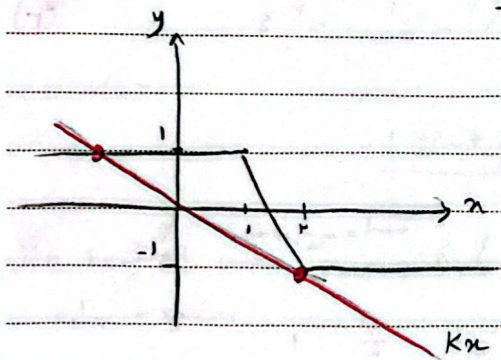
$\Rightarrow f(x) = \sqrt{bx+c} = 1 \quad \frac{r \cup \frac{1}{2}}{1} \quad bx+c=1 \quad (1) \quad bx-rb=1 \rightarrow \boxed{b=1} \quad \boxed{c=-r}$

$c = -rb$

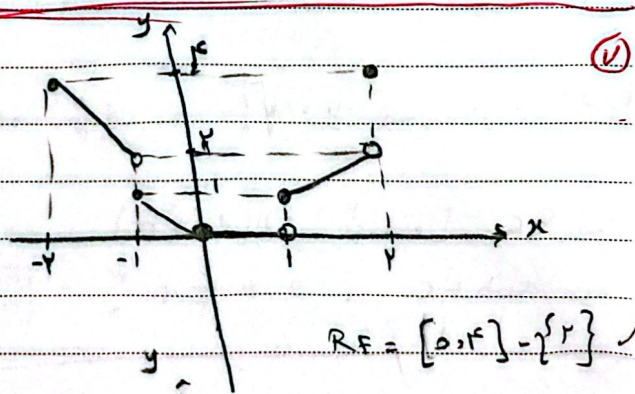
$g(x) = \sqrt{x^2 + 4} \xrightarrow{\omega_r} \omega_r, x \geq 0 \rightarrow R_f = \sqrt{[4, +\infty)} \rightarrow R_f = [2, +\infty) \checkmark$

$Kx = |x-2| - |x-1| \rightarrow \begin{array}{c} 1 \quad 2 \\ -x+2+x-1 \quad | \quad x-x-x+1 \\ \hline 1 \quad -x+1 \quad | \quad -x+1 \\ \hline 1 \quad -x+1 \quad | \quad -x+1 \end{array}$

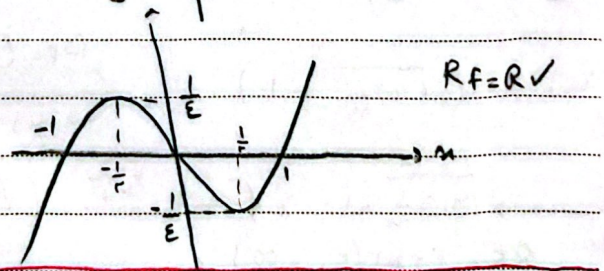
$\rightarrow K(2) = -1 \rightarrow \boxed{K = -\frac{1}{x}} \checkmark$



$\text{الف) } y = \begin{cases} -x & -2 \leq x < -1 \rightarrow [x] = -2 \\ -x & -1 \leq x < 0 \rightarrow [x] = -1 \\ 0 & 0 \leq x < 1 \rightarrow [x] = 0 \\ x & 1 \leq x < 2 \rightarrow [x] = 1 \\ \infty & x=2 \end{cases}$



$\text{ب) } y = x|x| - x \Rightarrow \begin{cases} x^2 - x & x > 0 \\ -x^2 - x & x < 0 \\ 0 & x = 0 \end{cases}$



$y = \frac{x+x+1}{x(x+1)} = \frac{2(x+1)}{x(x+1)} \xrightarrow{\text{لغز کثیر}} y = \frac{2}{x} \rightarrow y \neq 0$

$\Rightarrow R_f = R - \{0, -2\} \rightarrow a, b = 0, -2 \rightarrow a+b = -2+0 = \boxed{-2} \checkmark$

$$y = \frac{x - 4\sqrt{x} + 3}{x - 2\sqrt{x} + 1} \quad x > 0 \quad \sqrt{x} = t \quad y = \frac{t^2 - 4t + 3}{t^2 - 2t + 1} \quad (9)$$

$$y(t^2 - 2t + 1) = t^2 - 4t + 3 \Rightarrow (y-1)t^2 + (4-2y)t + y-3 = 0$$

(نیمه مثبت است)

$$x = \frac{2y - 4 + \sqrt{(4-2y)^2 - 4(y-1)(y-3)}}{2(y-1)} \quad \Delta = 4y^2 - 14y + 14 - 4(y^2 - 4y + 3) \geq 0$$

$$\rightarrow 4y^2 - 14y + 14 - 4y^2 + 16y - 12 \geq 0 \rightarrow 2y \geq 0 \rightarrow y \geq 0 \quad \checkmark$$

$$\rightarrow R_f = IR \quad \checkmark$$

$$y = \frac{x^3 + 2x^2 - x - 2}{x^2 - 1} \quad \text{صورت را اینجوری می‌نویسند: } \frac{x^3 + 2x^2 - x - 2}{x^2 - 1} \quad (10)$$

$$\Rightarrow y = \frac{(x+1)(x+2)(x-1)}{(x-1)(x+1)} = x+2$$

همه جا تقسیم می‌کنیم. x به ازای 1 و -1 تعریف نمی‌شود.

x به ازای 1 و -1 تعریف نمی‌شود. $y \neq 3$ و $y \neq 1$ $\rightarrow y = x+2$ $\rightarrow x \neq 1$ و -1

$$\rightarrow R_f = IR - \{3, 1\} \rightarrow 3+1 = 4 \quad \checkmark$$