به نام خدا

ماهان نژادکرد

۲۰ آفرین خیلی کامل نوشتی!

دوازدهم ریاضی B — تکلیف ۴

۱. (۲)

الف) $y=(x-2[\frac{x}{2}])^2=(2(\frac{x}{2})-2[\frac{x}{2}])^2$

$0 \le t-[t]<1 \Rightarrow 0 \le \frac{x}{2}-[\frac{x}{2}]<1$

$0 \le 2(\frac{x}{2})-2[\frac{x}{2}]<2 \Rightarrow 0 \le (2(\frac{x}{2})-2[\frac{x}{2}])^2<4$

$\Rightarrow R_f=[0,4)$

ب) $-\sin x+\log y-4=0 \Rightarrow \log y=\sin x+4 \Rightarrow y=10^{\sin x+4}$ $\quad \boxed{y>0}$ I

$-1 \le \sin x \le 1 \Rightarrow 3 \le \sin x+4 \le 5 \Rightarrow 10^3 \le 10^{\sin x+4} \le 10^5 \Rightarrow 10^3 \le y \le 10^5$ II

$\xrightarrow{I, II} R_f=[10^3,10^5]$

۲. (۲)

$y=\sqrt{\frac{x^2-4}{x^2-9}} \Rightarrow y^2=\frac{x^2-4}{x^2-9} \Rightarrow x^2=\frac{9y^2-4}{y^2-1} \ge 0$ $\quad \boxed{y \ge 0}$ I

	-1		$-\frac{2}{3}$		$\frac{2}{3}$		1	
$+$		$-$	0	$+$	0	$-$		$+$

$\Rightarrow (-\infty,-1)\cup[-\frac{2}{3},\frac{2}{3}]\cup(1,+\infty)$ II

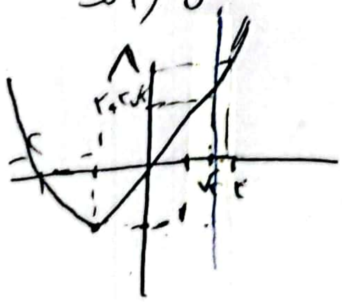
$\xrightarrow{I, II} R_f=[0,\frac{2}{3}]\cup(1,+\infty) \Rightarrow \begin{cases} a=0 \\ b=\frac{2}{3} \\ c=1 \end{cases} \Rightarrow \boxed{a+b+c=\frac{5}{3}}$

۳. (۲)

الف) $y=x+\frac{1}{x}+2\sqrt{x+\frac{1}{x}} \xrightarrow[x+\frac{1}{x} \ge 2]{\sqrt{x+\frac{1}{x}}=t} t^2+2t=y$

زیرا $\sqrt{2} \le t$

$R_f=[2+2\sqrt{2},+\infty)$



ب) $y = (3 - \sin x)(1 + \sin x) = -\sin^2 x + 2\sin x + 3$

$= -(\sin x - 1)^2 + 4 \Rightarrow \begin{cases} \sin x = 1 \Rightarrow y_{max} = 4 \\ \sin x = -1 \Rightarrow y_{min} = 0 \end{cases}$

$R_y = [0, 4]$

$y = -\frac{1}{2}x + 10$

ج- (۴)

$D_f = \{x \mid x \in \mathbb{N}, x < 10\} = \{1, 2, 3, \dots, 9\}$

$f(1) = 10 - \frac{1}{2}$

$f(2) = 10 - \frac{2}{2}$

$f(3) = 10 - \frac{3}{2}$

$\vdots$

$f(9) = 10 - \frac{9}{2}$

$f(1) + f(2) + f(3) + \dots + f(9) = 9 \times 10 - \frac{\frac{10 \times 9}{2}}{3} = 90 - 15 = 75$

د- (۲)

$f(x) = \sqrt{ax^2 + bx + c} \Rightarrow ax^2 + bx + c \geq 0$

$\begin{array}{c|c} & 2 \\ - & + \end{array}$

$\boxed{a = 0}$ تعیین علامت معادله به صورت خطی است پس ضریب x^2 برابر صفر است:

$bx + c \geq 0,\ b > 0$; $x = 2 \Rightarrow 2b + c = 0$; $x = 3 \Rightarrow 3b + c = 1$ $\Rightarrow \boxed{b = 1},\ \boxed{c = -2}$

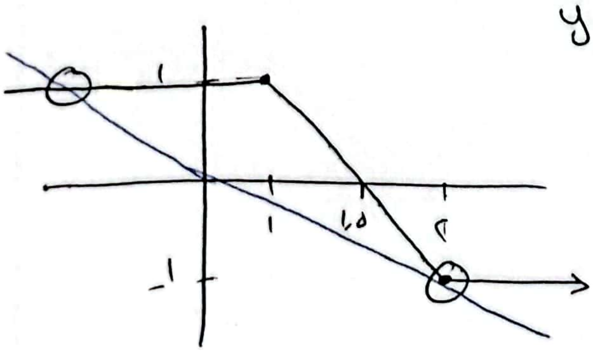
$g(x) = y = \sqrt{x^2 + 4} \Rightarrow x^2 + 4 \geq 4 \Rightarrow \sqrt{x^2 + 4} \geq 2 \Rightarrow R_f = [2, +\infty)$

ه- (۲)

$kx = |x - 2| - |x - 1|$

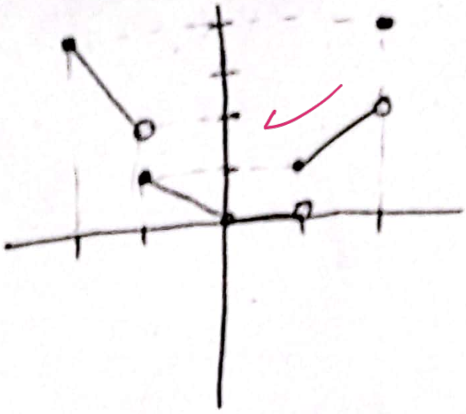
$y = kx \xrightarrow{x = 2} -1 = 2k \Rightarrow \boxed{k = -\frac{1}{2}}$

$y = -\frac{x}{2}$



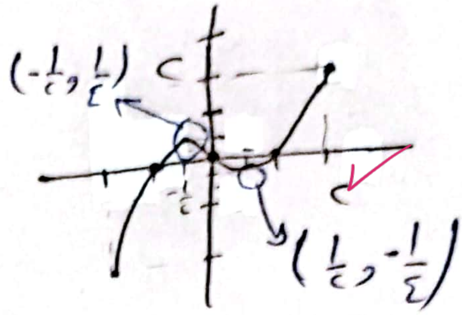
۲ - ۷

الف) $y = x[x] = \begin{cases} -2x & -2 \le x < -1 \\ -x & -1 \le x < 0 \\ 0 & 0 \le x < 1 \\ x & 1 \le x < 2 \\ 4 & x = 2 \end{cases}$



$R_f = [0, 4] - \{2\}$

ب) $y = x|x| - x = \begin{cases} -x^2 - x & -2 \le x < 0 \\ x^2 - x & 0 \le x \le 2 \end{cases}$



$R_f = [-2, 2]$

۲ - ۸

$y = \frac{1}{x} + \frac{1}{x+1} + \frac{1}{x^2+x} = \frac{1+x+x+1}{x^2+x} = \frac{2x+2}{x^2+x}$

$y = \frac{2}{x} \to R_f = \mathbb{R} - \{0\}$, $x \neq -1$ $\Big\} \Rightarrow R_f = \mathbb{R} - \{-2, 0\}$

$\boxed{a + b = -2 + 0 = -2}$

۲ - ۹

$y = \frac{x - 4\sqrt{x} + 3}{x - 2\sqrt{x} + 1} = \frac{(\sqrt{x}-1)(\sqrt{x}-3)}{(\sqrt{x}-1)^2} = \frac{\sqrt{x}-3}{\sqrt{x}-1}$

$x \neq 1$

$\to \sqrt{x} = \frac{y-3}{y-1} \ge 0$ — 1, 3: $+ \;|\; - \;|\; +$ $\Rightarrow \boxed{(-\infty, 1) \cup [3, +\infty) = R_f}$

۲ - ۱۰

$y = \frac{x^3 + 2x^2 - x - 2}{x^2 - 1} = \frac{\cancel{(x^2-1)}(x+2)}{\cancel{x^2-1}} \xRightarrow{x \neq \pm 1} y = x + 2 \Rightarrow \mathbb{R}$

$y \neq 1 + 2 = 3$

$y \neq -1 + 2 = 1$

$\boxed{3 + 1 = 4}$

$\to R_f = \mathbb{R} - \{3, 1\}$