

الف) $y = (x - 2(\frac{x}{y}))^2$
 $\Rightarrow (\frac{x}{y} - (\frac{x}{y}))^2 \rightarrow R_y = 4x^2$
 $\Rightarrow R_x = [0, 1) \Rightarrow R_x^2 = [0, 1)$
 $R_{xy} = [0, 2)$ ✓

ب) $- \sin u + \log y - 2 = 0$
 $\Rightarrow \log y = \sin u + 2 \Rightarrow y = 10^{\sin u + 2}$
 $-1 \leq \sin u \leq 1 \Rightarrow 1 \leq \sin u + 2 \leq 3$
 $\Rightarrow 10^1 \leq 10^{\sin u + 2} \leq 10^3 \Rightarrow R_y = [10, 10^3]$ ✓

$f(x) = \sqrt{\frac{x^2-4}{x^2-9}} \Rightarrow y^2 = \frac{x^2-4}{x^2-9} \Rightarrow x^2 = \frac{9y^2-4}{y^2-1} \Rightarrow x = \sqrt{\frac{9y^2-4}{y^2-1}}$
 $y \neq 1, -1$
 $\frac{y-1}{y+1} = \frac{1}{y} \Rightarrow \frac{y^2-1}{y^2+1} = \frac{1}{y} \Rightarrow y^3 - y = 1$
 $y > 0$ (I) $\Rightarrow R_f = [0, \frac{1}{y}] \cup (1, +\infty) \Rightarrow a=0, b=\frac{1}{y}, c=1$
 $\Rightarrow |ax+b+c| = 1 + \frac{1}{y} + 0 = \frac{y+1}{y}$ ✓

الف) $x = a + \frac{1}{a} + 2\sqrt{a + \frac{1}{a}}$
 $\Rightarrow \sqrt{a + \frac{1}{a}} = z \Rightarrow z^2 + 2z$
 $\min z = \min \sqrt{a + \frac{1}{a}} = \sqrt{2}$
 $\Rightarrow R_f = [2 + \sqrt{2}, +\infty)$ ✓

ب) $y = (3 - \sin u)(1 + \sin u)$
 $\Rightarrow y = 3 + 2\sin u - \sin^2 u$
 $\begin{cases} \sin u = -1 \\ \sin u = 1 \\ -\frac{b}{2a} = 1 \end{cases}$
 $\sin u = 1 \Rightarrow 3 + 2 - 1 = 4$
 $\sin u = -1 \Rightarrow 3 - 2 - 1 = 0$
 $\Rightarrow R_f = [0, 4]$ ✓

$f(x) = -\frac{1}{x}x + 10$ $f(1) = \frac{19}{x}$ $f(x) = \frac{11}{x}$ $f(x) = \frac{14}{x}$
 $D_f = \{x \mid x \in \mathbb{N}, x < 10\}$ $f(9) = 10 - 9 = 1 = \frac{1}{x}$
 $\Rightarrow S = \frac{n}{x}(n_1 + n_9) \Rightarrow \frac{9}{x}(\frac{19}{x} + \frac{11}{x}) \Rightarrow \frac{9}{x} \times \frac{30}{x} = \frac{270}{x^2} \neq \frac{1}{x}$ ✓

$f(x) = \sqrt{ax^2 + bx + c} \Rightarrow R_f = [2, +\infty) \Rightarrow$ $\frac{1}{x} \leq \sqrt{ax^2 + bx + c}$
 $\Rightarrow a=0, b=1, c=-2$ $f(2)=0 \Rightarrow 2b+c=0$
 $f(3)=1 \Rightarrow 3b+c=1 \Rightarrow b=1, c=-2$
 $g(x) = \sqrt{x^2 + 2}$
 $\Rightarrow R_g = [2, +\infty)$ ✓

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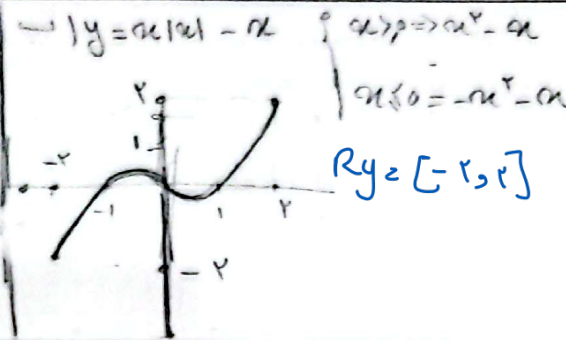
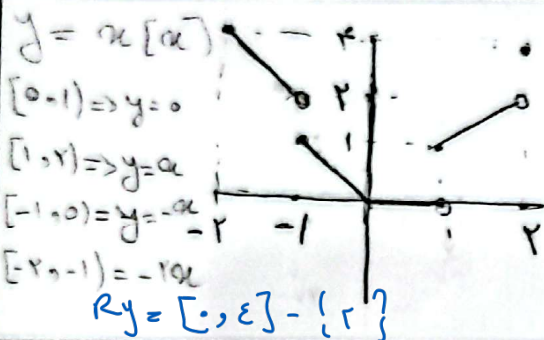
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$$|x-2| - |x-1| = kx$$

معادله خطی



نشان می دهد که جواب در $x=1$ و $x=2$ است
 اگر $x < 1$ و $x > 2$ داریم:
 $y = kx \Rightarrow y = -1, x = 2 \Rightarrow -1 = k \cdot 2 \Rightarrow 2k = -1$
 $\Rightarrow k = -\frac{1}{2}$



$$y = \frac{1}{x} + \frac{1}{x+1} + \frac{1}{x^2+x} \Rightarrow \frac{x+1+x+1}{x^2+x} \Rightarrow \frac{2x+2}{x^2+x} = y$$

$$y x^2 + y x = 2x + 2 \Rightarrow y x^2 + (y-2)x - 2 = 0 \Rightarrow \Delta = (y-2)^2 + 16y \geq 0$$

$$\Rightarrow y^2 - 4y + 4 + 16y \geq 0 \Rightarrow (y+2)^2 \geq 0 \Rightarrow y \geq -2$$

$$\Rightarrow y \neq -2, f(x) = \frac{2}{x} \Rightarrow f(x) \neq 0, y \neq 0 \Rightarrow y \neq -2 \Rightarrow 0 - 2 = -2$$

$$y = \frac{x - x\sqrt{x} + x^2}{x - x\sqrt{x} + 1} \Rightarrow \frac{(\sqrt{x} - 1)(\sqrt{x} - 1)}{(\sqrt{x} - 1)(\sqrt{x} - 1)} \Rightarrow \frac{\sqrt{x} - 1}{\sqrt{x} - 1} = y \Rightarrow \sqrt{x} = \frac{y-1}{y-1}$$

$$\Rightarrow \frac{1}{1+y} \leq \frac{1}{1+y} \Rightarrow R_f = (-\infty, 1) \cup [1, +\infty)$$

$$f(x) = \frac{x^2 + x^2 - x - 1}{x^2 - 1} \Rightarrow \frac{(x-1)(x+1)(x+1)}{(x-1)(x+1)}$$

$$\Rightarrow x=1 \Rightarrow 1+1=2$$

$$x=-1 \Rightarrow 1-1=0 \Rightarrow 1 \neq 0$$