①

الف) $y=(x-2[\frac{x}{2}])^2$ $\quad (\frac{2x}{2}-2[\frac{x}{2}])^2=(2(\frac{x}{2}-[\frac{x}{2}]))^2$

$$0\leq x-2[\tfrac{x}{2}]<4 \longleftarrow 0\leq \tfrac{x}{2}-[\tfrac{x}{2}]<1$$

ب) $-\sin x+\log y-4=0 \longrightarrow \log y=\sin x+4 \xrightarrow{-1\leq \sin x\leq 1} 3\leq \log y\leq 5 \longrightarrow R_f=[10^3,10^5]$

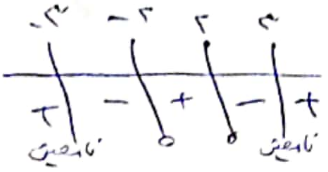
$[10^3,10^5]$

---

②

$f(x)=\sqrt{\frac{x^2-4}{x^2-9}}$ $\quad R_f=[a,b]\cup(c,+\infty)$ $\quad a+b+c=?$ $\quad \frac{5}{3}$

$\sqrt{\frac{(x-2)(x+2)}{(x-3)(x+3)}}$



$+\infty$
~~$\pm\infty$~~ $\longrightarrow$ ~~[illegible]~~ 1

$0\longrightarrow \frac{2}{3}$

$3^+\longrightarrow +\infty$ $\quad R=[0,\frac{2}{3}]\cup(1,+\infty)$

$2\longrightarrow 0$

$0+\frac{2}{3}+1=\frac{5}{3}$

---

③

الف) $f(x)=x+\frac{1}{x}+2\sqrt{x+\frac{1}{x}}$

ب) $(3-\sin x)(1+\sin x)=-\sin^2 x+2\sin x+3$

$\sin x=0\longrightarrow 3$ $\quad \sin x=1\longrightarrow 4$ $\quad \sin x=-1\longrightarrow 0$ $\quad R_f=[0,4]$ $\quad [0,4]$

---

④

$f(x)=-\frac{1}{3}x+10$ $\quad D_f=\{1,2,3,\dots,9\}$ $\quad S=(\frac{29}{3}+\frac{21}{3})\times\frac{9}{2}=\frac{50}{3}\times\frac{9}{2}=75$

$1\longrightarrow\frac{29}{3}$ $\quad 2\longrightarrow\frac{28}{3}$ $\quad 3\longrightarrow\frac{27}{3}$ $\quad \dots \quad 9\longrightarrow\frac{21}{3}$

$75$

---

⑤

$f(x)=\sqrt{ax^2+bx+c}$ $\quad D_f=[2,+\infty)$ $\quad f(2)=1$ $\quad g(x)=\sqrt{bx^2+ax\ \ -2c}$

⑥

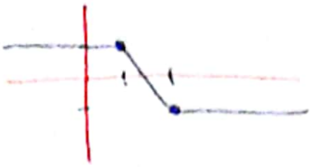
$|x-2|-|x-1|=kx$

$x=2 \qquad x=1$

$2 \to -1=kx \qquad kx=f(x)$

$1 \to 1=kx \qquad f(0)=0$

$0 \to 1=kx \qquad f(2)=1$



$k=\frac{1}{2}$

$\frac{1}{2}$

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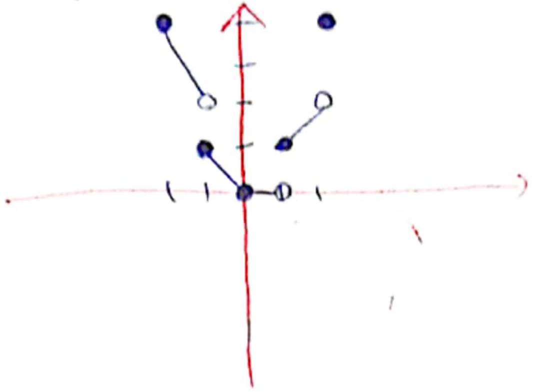
⑦ بازه $[-2,2]$

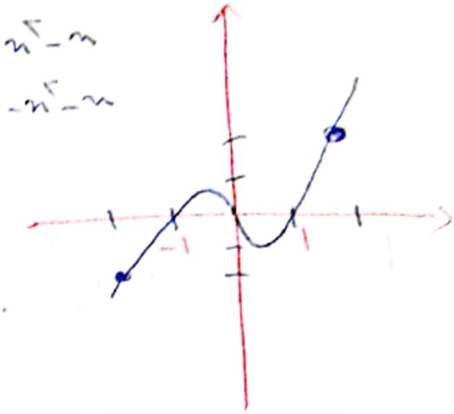
الف) $y=x[x]$



ب) $x|x|-x$

$x\geq 0 \to x^2-x$

$x<0 \to -x^2-x$



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⑧

$y=\frac{1}{x}+\frac{1}{x+1}+\frac{1}{\cancel{x^2+1}}$ $x(x+1$ $\qquad R_f=R-\{a,b\} \qquad a+b=?$

$\frac{x+1+x+1}{x(x+1)}=\frac{2x+2}{x(x+1)}=\frac{2\cancel{(x+1)}}{x\cancel{(x+1)}}\to\frac{2}{x}$

$\longrightarrow R=R-\{0,-1\}$ بخاطر اینه $(x+1)$ $\qquad a+b=-1$

$-1$

---

⑨

$f(x)=\frac{x-4\sqrt{x}+3}{x-2\sqrt{x}+1}\;\frac{(\sqrt{x}-3)\cancel{(\sqrt{x}-1)}}{(\sqrt{x}-1)^{\cancel{2}}}=\frac{(\sqrt{x}-3)}{(\sqrt{x}-1)}$ $\quad 0\to 3$, $+\infty\to 1$

مخرج صفر می‌شود

$(-\infty,1)\cup[3,+\infty)$ $\qquad (-\infty,1)\cup[3,+\infty)$

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⑩

$f(x)=\frac{x^3+2x^2-x-2}{x^2-1} \qquad R_f=R-\{a,b\} \qquad \frac{(x-1)(x^2+3x+2)}{\cancel{(x-1)}(x+1)}$

$\frac{(x+1)(x+2)}{x+1}=x+2\longrightarrow R \quad\longrightarrow R-\{1,3\}$

$~~D_f=\pm 1~~ \longrightarrow ~~R-\{1,3\}~~$

$R-\{1,3\}$

دوازده و نیم

مریم جانفزا