

$$y = \left(2\left(\frac{x}{2} - \left[\frac{x}{2}\right]\right)\right)^2 \rightarrow a - [a] \in [0, 1) \quad (الف)$$

$$\rightarrow 2\left(\frac{x}{2} - \left[\frac{x}{2}\right]\right) \in [0, 1) \rightarrow x \in [0, 2)$$

$$\rightarrow R_f = [0, 1)$$

$$\log y = \sin x + \pi \rightarrow \sin x + \pi \in [\pi/2, 3\pi/2] \quad (ب)$$

$$\rightarrow \log y \in [\pi/2, 3\pi/2] \rightarrow y \in [1, 10]$$

$$\rightarrow R_f = [1, 10]$$

$$y = \frac{x^2 - \pi}{x^2 - 9} \quad \begin{matrix} \text{max } x^2 = +\infty \rightarrow y = 1 \\ \text{min } x^2 = 0 \rightarrow y = \frac{\pi}{9} \end{matrix}$$

$$\Rightarrow R_g = (-\infty, \frac{\pi}{9}] \cup (1, +\infty)$$

$$\sqrt{R_g} = R_f = \left[\frac{\pi}{9}, 1\right] \cup (1, +\infty) \rightarrow a + b + c = 0 + \frac{\pi}{9} + 1 = \frac{\pi}{9} + 1$$

$$x + \frac{1}{x} = a + \frac{1}{a} \rightarrow \sqrt{x + \frac{1}{x}} \rightarrow x + \frac{1}{x} > 0 \rightarrow x + \frac{1}{x} \geq 2 \rightarrow \begin{matrix} \text{کمترین مقدار } x + \frac{1}{x} \text{ تابع ایلرا الصوری} \\ \text{کمترین مقدار آن ها در } x=1 \text{ و } x=-1 \text{ است} \\ \text{مقدار نزدیک به صفر نیستی} \end{matrix}$$

$$\rightarrow R_f = [2, +\infty)$$

$$(3 - \sin x)(1 + \sin x) = 3 + 2 \sin x - \sin^2 x$$

$$\rightarrow -\sin^2 x + 2 \sin x + 3 \quad \begin{matrix} -1 \rightarrow y = -1 + 2 + 3 = 4 \\ 1 \rightarrow y = -1 + 2 + 3 = 4 \\ -\frac{1}{2} \rightarrow y = -\frac{1}{4} + 1 + 3 = 3\frac{3}{4} \end{matrix}$$

$$\Rightarrow R_f = [0, 4]$$

$$D_f = \{1, 2, 3, \dots, 10, 9\} \rightarrow n(D) = 9 \rightarrow y_1 + y_2 + \dots + y_9 = 90 + (-\frac{1}{9})(1 + 2 + 3 + \dots + 9)$$

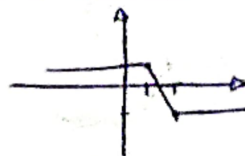
$$\rightarrow = 90 + (-\frac{1}{9})(45) = 90 - 5 = 85 \checkmark$$

$$f(x) = \sqrt{2bx + c} = 1 \rightarrow 2bx + c = 1 \quad (1)$$

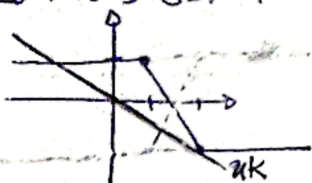
$$b > 0 \rightarrow \text{چون تابع صعودی است} \rightarrow f(x) = 0 \rightarrow \sqrt{2bx + c} = 0 \rightarrow 2bx + c = 0 \quad (2)$$

$$\text{نتیجه} \rightarrow b = 1, c = -2 \rightarrow g(x) = \sqrt{x^2 + \pi} \rightarrow x^2 > 0 \rightarrow \begin{matrix} \text{min } x^2 = 0 \\ \text{max } x^2 = +\infty \end{matrix} \rightarrow R_g = [\sqrt{\pi}, +\infty)$$

$$|x-2| - |x-1| = f(x) \Rightarrow f(1) = 1 \Rightarrow \text{تابع نزولی} \rightarrow$$



$\Rightarrow k \in \mathbb{R}$ شیبهای که در یک راستا باشند = از بعد ابعادی کمتر
مانند شکل زیر است.

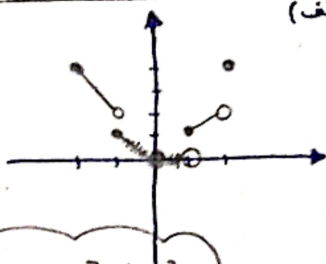


$$\rightarrow f(x) = 2k \cdot x - 1$$

$$\rightarrow k = \frac{-1}{2}$$

(ب)

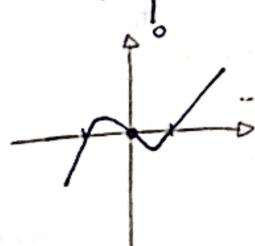
$$\begin{aligned} [-2, 0] &\rightarrow y = -2x \\ [0, 1] &\rightarrow y = -x \\ [1, 2] &\rightarrow y = 0 \\ [2, 3] &\rightarrow y = x \\ \{3\} &\rightarrow y = 3 \end{aligned}$$



$$\rightarrow R_f = [-2, 2] \cup \{3\}$$

(الف)

$$-x^2 - x = -x(x+1) \quad x^2 - x = x(x-1)$$



$$\rightarrow R_f = (-\infty, +\infty)$$

$$y = \frac{1}{x} + \frac{1}{x+1} + \frac{1}{x(x+1)} = \frac{x+1+x+1}{x(x+1)} = \frac{2(x+1)}{x(x+1)} \quad x \neq -1 \rightarrow y = \frac{2}{x}$$



$$\rightarrow y \in \mathbb{R} - \{f(-1), 0\} = \mathbb{R} - \{-2, 0\}$$

$$\rightarrow -2 + 0 = -2$$

$$f(x) = \frac{x - \sqrt{x} + 1}{x - 2\sqrt{x} + 1} = \frac{(\sqrt{x}-1)^2 - 1}{(\sqrt{x}-1)^2} = \frac{(\sqrt{x}-1)(\sqrt{x}-2)}{(\sqrt{x}-1)^2} = \frac{(\sqrt{x}-2)}{(\sqrt{x}-1)}$$

خرج ریشه مضامف
داره بی لازم نیست لحاظ کنیم!

$$= \frac{\sqrt{x}-2}{\sqrt{x}-1} = y \rightarrow$$

$$\begin{aligned} \max \sqrt{x} &\rightarrow y = 1 \\ \min \sqrt{x} &= 0 \rightarrow y = 2 \end{aligned}$$

$\Rightarrow$ هیچ منفی $\Rightarrow$ خارج

$$\rightarrow R_f = (-\infty, 1) \cup [2, +\infty)$$

$$f(x) = \frac{x^3 + x^2 + x - x - 2}{(x-1)(x+1)} = \frac{x^3 + x^2 + (x-2)}{(x-1)(x+1)} \xrightarrow{x \neq -1} \frac{x^3 + x - 2}{x-1}$$

$$= \frac{(x+2)(x-1)}{(x-1)} \xrightarrow{x \neq 1} y = x+2 \rightarrow R_f = \mathbb{R} - \{f(1)\} = \mathbb{R} - \{3\}$$

$$\rightarrow 3+1=4$$