امیرمحمد ابوطالبی

۱- ۱/۷۵

الف) $y=(x-2[\frac{x}{2}])^2 \longrightarrow y=2(\frac{x}{2}-[\frac{x}{2}])^2 \xrightarrow{17/25} R_y=[0,2)$

$[0,1) \to \times 2 \to [0,2) \xrightarrow{\text{توان}2} [0,4)$

ب) $y=-\sin x+\log y-4= \longrightarrow \underset{[-1,1]}{\sin x}=\log y-4 \longrightarrow -1\leq \log y-4\leq 1 \longrightarrow 3\leq \log y\leq 5$

$\longrightarrow R_y=[10^3,10^5]$

۲- ۲

$f(x)=\sqrt{\frac{x^2-4}{x^2-9}} \longrightarrow y=\sqrt{\frac{x^2-4}{x^2-9}} \longrightarrow y^2=\frac{x^2-4}{x^2-9} \to y^2x^2-9y^2=x^2-4$

$y^2x^2-x^2=9y^2-4 \longrightarrow x^2(y^2-1)=9y^2-4 \longrightarrow x^2=\frac{9y^2-4}{y^2-1} \longrightarrow x=\sqrt{\frac{9y^2-4}{y^2-1}}$

$\frac{9y^2-4}{y^2-1}\geq 0 \longrightarrow \frac{(3y-2)(3y+2)}{(y-1)(y+1)}\geq 0 \longrightarrow$ + -1 - -2/3 + 2/3 - 1 + $\to R_y=[0,\frac{2}{3}]\cup(1,+\infty)$

$\frac{2}{3}+0+1=\frac{5}{3}$

۳- ۲

الف) $f(x)=x+\frac{1}{x}+2\sqrt{x+\frac{1}{x}} \longrightarrow x+\frac{1}{x}\geq 0 \longrightarrow \frac{x^2+1}{x}\geq 0 \longrightarrow x>0$

if $a>0 \longrightarrow a+\frac{1}{a}\geq 2 \longrightarrow [2+2\sqrt{2},+\infty)$

ب) $\longrightarrow y=(3-\sin x)(1+\sin x) \longrightarrow y=-\sin^2x+2\sin x+3 \to$

$\sin x=1 \to y=4$

$\sin x=-1 \to y=0$

$\frac{-b}{2a}=1 \to y=4$

$[0,4]$

۴- ۲

$D_f=\{1,2,3,4,5,6,7,8,9\}$

$R_f=\{\frac{29}{3},\frac{28}{3},\frac{27}{3},\frac{26}{3},\frac{25}{3},\frac{24}{3},\frac{23}{3},\frac{22}{3},\frac{21}{3}\} \longrightarrow \frac{29}{3}+\frac{28}{3}+\frac{27}{3}+\dots+\frac{21}{3}=\frac{225}{3}$ ۷۵

$=75$

۵- ۰

~~$f(x)=\sqrt{cx^2+bx+c}$ ... $[2,+\infty)$ ... $y=\sqrt{a(x-2)^2}$ ...~~

~~$1=\sqrt{a} \longrightarrow \boxed{a=1},\boxed{b=-4},\boxed{c=4}$~~ $\longrightarrow \boxed{a=1},\boxed{b=-4},\boxed{c=4}$

دامنه نشان می‌دهد تابع خطی است $\longleftarrow a=0$

$f(3)=2$, $f(2)=0 \longrightarrow b=1$, $c=-2$

$g(x)=\sqrt{x^2+4}$

$R_g=[2,+\infty)$

~~$g(x)=\sqrt{-4x^2+x-8}$~~

$g(x)=\sqrt{-4x^2+x-8} \longrightarrow R_g=\varnothing$

۶- ۲

$|x-2|-|x-1|=kx \longrightarrow$

$x\geq 2 \longrightarrow -x-2-x+1=kx \to kx=-1$

$1\leq x<2 \to -x+2-x+1=kx \to (k+2)x=3$

$x<1 \to -x+2+x-1=kx \to kx=1$

شکل رسم کن!

با امتحان $\longrightarrow$ ~~$\boxed{k=-\frac{1}{2}}$ ... $x=-2$~~

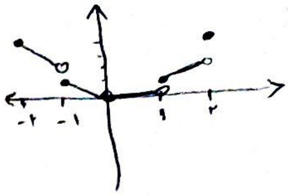
~~$(-\frac{1}{2}+2)x=3 \to \frac{3}{2}x=3 \to x=2$~~ X

$k=-\frac{1}{2} \to x=2,-2$

۷ـ ۱/۵

الف) $x[x]$

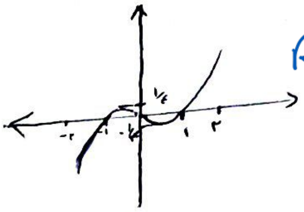
$x=2 \to 2x$
$1\le x<2 \to x$
$0\le x<1 \to 0$
$-1\le x<0 \to -x$
$-2\le x<-1 \to -2x$



$R_y=[0,4]-\{2\}$

ب) $x|x|-x \to$

$x\ge 0 \to x^2-x$
$x<0 \to -x^2-x$



$R_y=[-2,2]$

۸ـ ۲

$y=\frac{1}{x}+\frac{1}{x+1}+\frac{1}{x^2+x} \to y=\frac{x+1}{x^2+x}+\frac{x}{x^2+x}+\frac{1}{x^2+x}$

$y=\frac{2x+2}{x^2+x} \to R_y=R-\{0,-2\} \to \boxed{0+-2=-2}$

۹ـ ۲

$\cancel{x=1}\ y=\frac{(\sqrt{x}-2)^2-1}{(\sqrt{x}-1)^2}=\frac{(\sqrt{x}-3)(\sqrt{x}-1)}{(\sqrt{x}-1)\cancel{(\sqrt{x}-1)}}=\frac{\sqrt{x}-3}{\sqrt{x}-1}$

$y=\frac{\sqrt{x}-3}{\sqrt{x}-1} \to y\sqrt{x}-y=\sqrt{x}-3 \to (y-1)\sqrt{x}=y-3 \to \sqrt{x}=\frac{y-3}{y-1}$

$\frac{y-3}{y-1}\ge 0 \to + \ 1 \ - \ 3 \ + \to$ ~~R-[1,3]~~ $R_y=R-[1,3)$

۱۰ـ ۲

$y=\frac{x^3+2x^2-x-2}{x^2-1}=\frac{(x^2-1)(x+2)}{(x^2-1)} \to R_y=R-\{1,3\}$

$\boxed{3+1=4}$