امیرمحمد ابوطالبی

۱- الف) $y=(x-2[\frac{x}{2}])^2 \longrightarrow y=2(\underbrace{\frac{x}{2}-[\frac{x}{2}]}_{[0,1)})^2 \longrightarrow R_y=[0,2)$

ب) $y=-\sin x+\log y-4= \longrightarrow \underbrace{\sin x}_{[-1,1]}=\log y-4 \longrightarrow -1\leq \log y-4\leq 1 \longrightarrow 3\leq \log y\leq 5$
$\longrightarrow R_y=[10^3,10^5]$

۲-

$f(x)=\sqrt{\frac{x^2-4}{x^2-9}} \longrightarrow y=\sqrt{\frac{x^2-4}{x^2-9}} \longrightarrow y^2=\frac{x^2-4}{x^2-9} \to y^2x^2-9y^2=x^2-4$

$y^2x^2-x^2=9y^2-4 \longrightarrow x^2(y^2-1)=9y^2-4 \to x^2=\frac{9y^2-4}{y^2-1} \to x=\sqrt{\frac{9y^2-4}{y^2-1}}$

$\frac{9y^2-4}{y^2-1}\geq 0 \longrightarrow \frac{(3y-2)(3y+2)}{(y-1)(y+1)}\geq 0 \longrightarrow \to R_y=(0,\frac{2}{3}]\cup[1,+\infty)$

$\frac{2}{3}+0+1=\frac{5}{3}$

۳- الف) $f(x)=x+\frac{1}{x}+2\sqrt{x+\frac{1}{x}} \longrightarrow x+\frac{1}{x}\geq 0 \longrightarrow \frac{x^2+1}{x}\geq 0 \longrightarrow x\geq 0$

if $a\geq 0 \to a+\frac{1}{a}\geq 2 \longrightarrow [2+2\sqrt{2},+\infty)$

ب) $y=(3-\sin x)(1+\sin x) \to y=-\sin^2 x+2\sin x+3 \to$
$\sin x=1 \to y=4$
$\sin x=-1 \to y=0$
$\frac{-b}{2a}=1 \to y=4$
$[0,4]$

۴-

$D_f=\{1,2,3,4,5,6,7,8,9\}$

$R_f=\{\frac{29}{3},\frac{28}{3},\frac{27}{3},\frac{26}{3},\frac{25}{3},\frac{24}{3},\frac{23}{3},\frac{22}{3},\frac{21}{3}\} \longrightarrow \frac{29}{3}+\frac{28}{3}+\frac{27}{3}+\dots+\frac{21}{3}=\frac{225}{3}$
$=75$

۵-

~~$f(x)=\sqrt{ax^2+bx+c}$ ... $[2,+\infty)$ ... $y=\sqrt{a(x-2)^2}$ ... $1=\sqrt{a} \to a=1,\ b=-4,\ c=4$~~ $\longrightarrow \boxed{a=1},\ \boxed{b=-4},\ \boxed{c=4}$

$g(x)=\sqrt{-4x^2+x-8}$

$g(x)=\sqrt{-4x^2+x-8} \longrightarrow R_g=\varnothing$

۶-

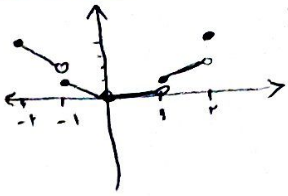
$|x-2|-|x-1|=kx \longrightarrow$
$x\geq 2 \longrightarrow x-2-x+1=kx \to kx=-1$
$1\leq x<2 \to -x+2-x+1=kx \to (k+2)x=3$
$x<1 \to -x+2+x-1=kx \to kx=1$

با امتحان $\longrightarrow$ ~~$k=-\frac{1}{2}$ ... $x=2$ ... $(-\frac{1}{2}+2)x=3 \to \frac{3}{2}x=3 \to x=2$~~ X

$k=-\frac{1}{2} \to x=2,-2$

الف) $x[x]$

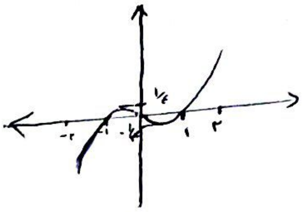
$x=۲ \rightarrow ۲x$
$۱\leq x<۲ \rightarrow x$
$۰\leq x<۱ \rightarrow ۰$
$-۱\leq x<۰ \rightarrow -x$
$-۲\leq x<-۱ \rightarrow -۲x$



۷-

ب) $x|x|-x \longrightarrow$

$x\geq ۰ \rightarrow x^۲-x$
$x<۰ \rightarrow -x^۲-x$



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$y=\frac{۱}{x}+\frac{۱}{x+۱}+\frac{۱}{x^۲+x} \longrightarrow y=\frac{x+۱}{x^۲+x}+\frac{x}{x^۲+x}+\frac{۱}{x^۲+x}$

$y=\frac{۲x+۲}{x^۲+x} \longrightarrow R_y=R-\{۰,-۲\} \longrightarrow ۰+-۲=-۲$

۹

$~~x=~~$ $y=\frac{(\sqrt{x}-۲)^۲-۱}{(\sqrt{x}-۱)^۲}=\frac{(\sqrt{x}-۳)(\sqrt{x}-۱)}{(\sqrt{x}-۱)\cancel{(\sqrt{x}-۱)}}=\frac{\sqrt{x}-۳}{\sqrt{x}-۱}$

$y=\frac{\sqrt{x}-۳}{\sqrt{x}-۱} \rightarrow y\sqrt{x}-y=\sqrt{x}-۳ \rightarrow (y-۱)\sqrt{x}=y-۳ \rightarrow \sqrt{x}=\frac{y-۳}{y-۱}$

$\frac{y-۳}{y-۱}\geq ۰ \longrightarrow + \;۱\; - \;۳\; + \longrightarrow$ ~~R-[1,3)~~ $R_y=R-[۱,۳)$

۱۰-

$y=\frac{x^۳+۲x^۲-x-۲}{x^۲-۱}=\frac{(x^۲-۱)(x+۲)}{(x^۲-۱)} \rightarrow R_y=R-\{۱,۳\}$

$x^۲-۱$

$\boxed{۳+۱=۴}$