امین جلالی

۲۰ آفرین !!

①

$D_{gof} = \{x \in D_f \mid f(x) \in D_g\}$

$D_f = \begin{cases} x-1>0 \to x>1 \ (\mathrm{I}) \\ \log_2^{x-1} \geq 0 \Rightarrow x-1 \geq 1 \Rightarrow x \geq 2 \ (\mathrm{II}) \end{cases} \Rightarrow D_f \xrightarrow{\mathrm{I}\cap\mathrm{II}} [2,+\infty)$

$D_{g(x)} \to -x^2+4x-4 \geq 0 \to x^2-4x+4 \leq 0 \to (x-2)^2 \leq 0 \Rightarrow x=2 \to D(gx)=2$

$D_{gof} = \{x \in [2,+\infty) \mid \sqrt{\log_2^{(x-1)}} = 2\} \Rightarrow D_{gof} = \{17\}$

2

②

$fog(x) = f(g(x)) \Longrightarrow 2(x^2-3x+8)-5 = \boxed{2x^2-6x+11}$

$gof(x) = g(f(x)) \Longrightarrow (2x-5)^2-3(2x-5)+8 = 4x^2+25-20x-6x+15+8 = \boxed{4x^2-26x+48}$

$\Longrightarrow 2x^2-6x+11 = 4x^2-26x+48 \Longrightarrow 2x^2-20x+37 \Rightarrow \alpha^2+\beta^2 = S^2-2P$

$\Rightarrow S = -\frac{b}{a} = \boxed{10} \ , \ \boxed{P = \frac{c}{a} = \frac{37}{2}}$

$\Longrightarrow 100-37 = \boxed{43}$

2

③

$f(x) = 2x \Rightarrow t = 2x \Rightarrow x = \frac{t}{2}$

$\Rightarrow g(t) = \frac{5}{4}t^2+11 \Rightarrow g(x) = \frac{5x^2}{4}+11$

$y = -\frac{\Delta}{4a} = -\frac{55}{-5} = \boxed{11}$

چون برد $g(x)=g(x-7)$ ⇐ فقط دامنه آنها متفاوت می باشد

⇐ کمترین مقدار $g(x)$ را بدست می آوریم. ⇐

2

④

$f(x) = 3x-4 \to f(g(x)) = 3g(x)-4 = 3x^2-6x-1 = fog(x) \to g(x) = x^2-2x+1$

$gof(x) = (3x-4)^2-2(3x-4)+1 = 9x^2-24x+16-6x+8+1 = \boxed{9x^2-30x+25}$

2

⑤

$f(x)-g(x) = -x+5 \to 2f(x)-2g(x) = -2x+10 \xrightarrow{2g(x)=3f(x)-14} 2f(x)-3f(x)+14 = -2x+10 \Rightarrow f(x) = 2x+4$

$2g(x) = 3f(x)-14 \to 2g(x) = 6x+12-14 \to g(x) = 3x-1 \quad fog(x) = f(g(x)) =$

$6x-2+4 = 9x+3 \Longrightarrow 6x+2 = 0 \to x = -\frac{1}{3} \Rightarrow \boxed{a = -\frac{1}{3}}$

2

$f\circ g(x) = f(g(x)) = 9^{\log_3 x} = x^2$ / $g\circ f(x) = g(f(x)) = \log_3 9^x = 2x$ (6)

$\Rightarrow x^2 \leq 2x \longrightarrow x^2 - 2x \leq 0 \longrightarrow \begin{array}{ccccc} & 0 & & 2 & \\ + & 0 & - & 0 & + \end{array} \Rightarrow x \in [0,2]$ (I)

$D_{f\circ g(x)} = \{x \in D_{g(x)} \mid g(x) \in D_{f(x)}\} = \{x > 0 \mid \log_3 x \in \mathbb{R}\} = (0,+\infty)$ (II) 2

$D_{g\circ f(x)} = \{x \in D_{f(x)} \mid f(x) \in D_{g(x)}\} = \{x \in \mathbb{R} \mid 9^x > 0\} = \mathbb{R}$ (III)

$\xrightarrow{\mathrm{I} \cap \mathrm{II} \cap \mathrm{III}} [0,2] \longrightarrow$ $\boxed{\text{گزینه ۳ صحیح}}$

$f\left(\frac{x^2-2}{x}\right) = x^2 + x - 4 - \frac{2}{x} + \frac{4}{x^2} = \frac{x^4 + x^3 - 4x^2 - 2x + 4}{x^2} = \frac{(x^2-2)^2 + x(x^2-2)}{x^2}$ (7) 2

$= \frac{(x^2-2)(x^2-2)}{x \times x} + \frac{x(x^2-2)}{x^2} \xrightarrow{\frac{x^2-2}{x} = t} f(t) = t^2 + t \Rightarrow f(x) = x^2 + x$

$D_{f\circ g} = \{x \in D_{g(x)} \mid g(x) \in D_{f(x)}\} \longrightarrow \{x \in \mathbb{R} \mid 8x - x^2 > 0 \to \begin{array}{cc} 0 & 8 \end{array}\}$ (8)

$\longrightarrow D_{f\circ g} = (0,8) = A$

$f\circ g(x) = \log_2 (8x - x^2) \longrightarrow$ چون برد $8x-x^2$ در بازهٔ $(0,8)$ بازه $(0,16]$ است و تابع‌ها در بازهٔ $(0,4]$ و $(4,8)$ اکیداً صعودی است با جایگذاری دامنه برد به دست می‌آید. 2

$x^+ \to 0 \quad \log_2 0^+ = -\infty$

$x \to 4 \quad \log_2 16 = 4$

$x \to 8^- \quad \log_2 0^+ = -\infty$

$\Big\} \to R_{f\circ g} = (-\infty, 4] = B \Rightarrow A \cap B = (0,4]$

$f\circ f(x) = 1 - 3f^2(x) \xrightarrow{f(x)=t} f(t) = 1 - 3t^2 \to f(x) = 1 - 3x^2$

$h(g(x)) = (x-1)^3 + 2(x-1) + 1 = x^3 - 3x^2 + 3x - 1 + 2x - 2 + 1 = x^3 - 3x^2 + 5x - 2$ (9) 2

$h(g(x)) = f(x) \longrightarrow x^3 - 3x^2 + 5x - 2 = 1 - 3x^2 \longrightarrow x^3 + 5x - 3 = 0 \longrightarrow$ چون درجه ۳ است نقطه اشتراک دارد

$f(x) = x - \frac{4}{x}$ (10) 2

$g(3) = 8 \longrightarrow g(3) = g(f(x)) \longrightarrow f(x) = 3 \longrightarrow x - \frac{4}{x} = 3 \longrightarrow x^2 - 3x - 4 = 0 \to x = -1,\ x = 4$

$\longrightarrow g(f(x)) = ax^3 + 2x \to g(f(-1)) = g(3) = -a - 2 = 8 \to a = 10$

$\to g(f(4)) = g(3) = 64a + 8 = 8 \to a = 0$