

$$g(x) = \sqrt{-x^2 + 2x - 1} = \sqrt{-(x^2 - 2x + 1)} = \sqrt{-(x-1)^2} \rightarrow x=1$$

$$f(x) = \sqrt[4]{x-1} \rightarrow x > 1 \quad \left. \begin{array}{l} f(x)=2 \\ \end{array} \right\} \leftarrow$$

$$\sqrt[4]{x-1} = 2 \rightarrow x-1 = 16 \rightarrow \boxed{x=17} \quad \checkmark$$

$$f \circ g(x) = 2x^2 - 4x + 11 = 5, \quad g \circ f(x) = (x-5)^2 - 4x + 10 = 1$$

$$\begin{array}{l} 2x^2 - 4x + 11 = 5 \\ 2x^2 - 4x + 6 = 0 \\ x^2 - 2x + 3 = 0 \end{array}$$

$$\Delta = 4 - 12 = -8 < 0 \quad \text{no real roots}$$

$$f \circ g(x) = 5 \rightarrow 2x^2 - 4x + 11 = 5 \rightarrow 2x^2 - 4x + 6 = 0 \rightarrow x^2 - 2x + 3 = 0$$

$$x = 1 \pm \sqrt{1-3} = 1 \pm \sqrt{-2} \quad \text{no real roots}$$

$$g(2x) = 2x^2 + 11 \rightarrow g(x) = \frac{1}{2}x^2 + 11 \rightarrow g(x-1) = \frac{1}{2}(x^2 - 2x + 1) + 11$$

$$= \frac{1}{2}x^2 - x + \frac{1}{2} + 11 = \frac{1}{2}x^2 - x + \frac{23}{2}$$

$$-\frac{b}{2a} = 1 \rightarrow \boxed{11} \quad \checkmark$$

$$f \circ g(x) = 3x^2 - 4x - 1 \rightarrow 3x^2 - 4x - 1 = 0 \rightarrow x = \frac{4 \pm \sqrt{16 + 12}}{6} = \frac{4 \pm 5}{3}$$

$$x = 3 \text{ or } x = -\frac{1}{3}$$

$$g \circ f(x) = (3x - 5)^2 = 9x^2 - 30x + 25$$

$$f \circ g(x) = 4x + 2 \Rightarrow \boxed{a = -\frac{1}{4}} \quad \checkmark$$

$$f \circ g(x) = -x + 5 \rightarrow g(x) = 13x + 1$$

$$2f(x) - 2g(x) = 12 - f(x) \rightarrow f(x) = 2x + 2$$

$$\begin{aligned} f \circ g &= g^{\frac{x}{r}} = x^r \\ g \circ f &= g^{\frac{x}{r}} = rx \end{aligned} \quad \left\{ \begin{array}{l} x^r, rx \rightarrow x^r - rx = 0 \\ \frac{x^r}{+ \cancel{rx} +} \end{array} \right\} \quad x \in [0, r]$$

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$$f\left(x - \frac{r}{x}\right) = \left(x - \frac{r}{x}\right)^r + \left(x - \frac{r}{x}\right) \rightarrow x - \frac{r}{x} = t$$

$$\underbrace{x^r - r + \frac{r}{x^r}}_{x^r - r + \frac{r}{x^r}} \quad f(t) = t^r + t$$

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$$\boxed{f(x) = x^r + x}$$

$$f \circ g(x) = g^{\frac{1}{r} \wedge x - x^r} \rightarrow D_f: \frac{1}{r} \wedge x - x^r > 0 \rightarrow D_f \in (0, 1) = A$$

$$\downarrow$$

$$\mathbb{R}_f: 0 < \frac{1}{r} \wedge x - x^r \leq 1 \Rightarrow \mathbb{R}_f \in (-\infty, r] = B$$

$$A \cap B \in (0, r] \rightarrow 1, r, r, r \rightarrow \text{مضرب}$$

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$$f \circ f(x) = 1 - r f^r(x) \Rightarrow f(x) = 1 - r x^r$$

$$h \circ g(x) = (x-1)^r + rx - 1 = 1 - r x^r \rightarrow x^r + ax - r = 0 \rightarrow \text{یک جواب}$$

$$f(x) \left\{ \begin{array}{l} x^r - r x^r + r x + r x - r \\ \quad \quad \quad + ax \end{array} \right.$$

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$$g(r) = 1 \rightarrow g(f(x)) = 1 \rightarrow f(x) = r \rightarrow x^r - rx - r = 0 = (x-r)(x+1) = 0$$

$$\begin{aligned} x = -1 &\rightarrow -a - r = 1 \rightarrow a = -1 - r \\ x = r &\rightarrow r(a+1) = 1 \rightarrow a = 0 \end{aligned}$$

$$\begin{aligned} x &= r \\ \leftarrow x &= -1 \end{aligned}$$

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