صالح ثنائی — بنام آنکه هستی نام از او یافت — ۱۸ — دوازدهم تیر B - تکلیف

① ۱/۵

$D_{gof} = \{x \in D_f : f(x) \in D_g\} \to D_{gof} : D_f = \begin{cases} x-1>0 \to x>1 \ (I) \\ \log_2^{(x-1)} \geq 0 \to x-1 \geq 1 \to x \geq 2 \end{cases}$

$D_g = -(x^2-4x+4) \geq 0 \to \underbrace{-(x-2)^2}_{\text{همواره نامثبت}} \geq 0 \to D_g = \{2\} \to f(x) = 2 \to \sqrt{\log_2^{x-1}} = 2$

$\to x-1 = 16 \to x = \cancel{5}\ 17 \ (III) \to D_{gof} = (I)\cap(II)\cap(III) = \{\cancel{5}\ 17\}$

$\sqrt{\log_2^{x-1}} = 2 \to \log_2^{x-1} = 4 \to x-1 = 16 \to x = 17$

②

$fog = 2(x^2-3x+8)-5 = 2x^2-6x+11$

$gof = (2x-5)^2-3(2x-5)+8 = 4x^2-26x+48$

$\Rightarrow 2x^2-6x+11 = 4x^2-26x+48 \to 2x^2-20x+37=0 \to S=10 \quad P=\frac{37}{2}$

$\alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta = S^2-2P = 100-37 = 63$

③ ۱/۵

$gof = g(f(x)) = g(2x) = 5x^2+11 \to 2x = t \to x = \frac{t}{2} \to g(t) = 5\left(\frac{t}{2}\right)^2+11$

$= \frac{5t^2}{4}+11 \to g(x) = \frac{5x^2}{4}+11 \to g(x-7) = \frac{5(x-7)^2}{4}+11 = \frac{5x^2}{4}-\frac{70x}{4}+\frac{289}{4}$

$R_g = [11, +\infty) \longrightarrow R_{g(x-7)} = [11, +\infty)$

$x_S = \frac{-b}{2a} = \frac{\frac{70}{4}}{\frac{10}{4}} = 7 \to y_S = \frac{44}{11} = 4$

انتقال افقی تاثیری در برد تابع ندارد.

④

$f(g(x)) = 3g(x)-4 = 3x^2-6x-1 \to g(x) = x^2-2x+1 \to gof = (3x-4)^2-2(3x-4)+1$

$9x^2+16-24x-6x+8+1 = 9x^2-30x+25$

۲

⑤ ۲

$g(x) = \frac{3f(x)}{2}-7 \ , \ f-g = -x+5 = f(x)-\frac{3f(x)}{2}+7 \to f(x) = 2x+4$

$g(x) = \frac{3}{2}(2x+4)+7 = 3x-1 \to fog = 2(3x-1)+4 = 6x+2 \to \Big|_0^a \to a = -\frac{1}{3}$

⑥

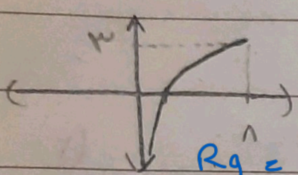
$fog = 9^{\log_3^x} = x^2 \ , \ gof = \log_3^{9^x} = 2\log_3^{3^x} = 2x \to 2x \geq x^2$

۲

$2x - x^2 \geq 0$ $\quad \begin{array}{c|c|c} & 0 & 2 \\ - & + & - \end{array} \to x = [0, 2] \to$ ۳ عدد صحیح

$\left(x - \frac{2}{x}\right)^2 = x^2 + \frac{4}{x^2} - 4 \to f\left(x - \frac{2}{x}\right) = \left(x - \frac{2}{x}\right)^2 + \left(x - \frac{2}{x}\right) \to f(x) = x^2 + x$ ۷ ۴

$fog = \log_2^{8x - x^2} \to D_{fog} = 8x - x^2 > 0 \to \begin{array}{c|c|c} & 0 & 8 \\ - & + & - \end{array} \to D_{fog} = (0, 8)$ ۸ ۰٫۲۵




$R_{fog} = (-\infty, 4) \to R \cap D_{fog} = (0, 4] \to$ شامل ۴ عدد صحیح

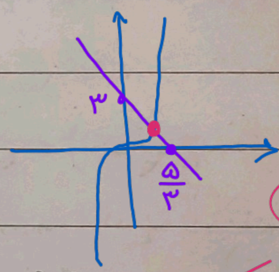
$R_{g} = (-\infty, 16] \xrightarrow{\log_2^x} (-\infty, 4] \to A \cap B = (0, 4]$

$fof = f(f(x)) = 1 - 3f^2(x) \to f(x) = 1 - 3x^2$ ۱٫۵ ۹

$h(g(x)) = (x-1)^3 + 2(x-1) + 1 = 1 - 3x^2 \to x^3 - 3x^2 + 5x - 2 = 1 - 3x^2$

$\to x^3 + 5x - 3 = 0 \to$ ۳ جواب دارد $\quad x^3 = -5x + 3$

تک جوابه ★




۲

$x - \frac{4}{x} = 3 \to x^2 - 3x - 4 \to x = -1, 4 \to f\left(x - \frac{4}{x}\right) \overset{x=4}{=} 8 = 64a + 8 \to a = 0$ ۱۰

$\to f\left(x - \frac{4}{x}\right) \overset{x=-1}{=} -a - c = 8 \to a = -10$