

نام و نام خانوادگی: پاسخنامه تشریحی شماره ۱. کلاس: B

$$D_f = x > 1$$

$$D_g = \emptyset$$

$$g(x) = \sqrt{-(x-2)^2}$$

مجموعه تهی

$$D_{g \circ f} = \{x \in D_f : f(x) \in D_g\} \Rightarrow D_{g \circ f} = \emptyset$$

$x > 1$ $f(x) \in \emptyset$

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$$f \circ g(x) = 2(x^2 - 4x + 1) - 6 = 2x^2 - 4x + 1$$

$$g \circ f(x) = 2(x-6)^2 - 2(x-6) + 1 = 2x^2 - 20x + 26 - 4x + 12 + 1 = 2x^2 - 24x + 39$$

$$2x^2 - 4x + 1 = 2x^2 - 24x + 39 \Rightarrow 2x^2 - 20x + 38 = 0$$

$$\alpha^2 + \beta^2 \leq 5^2 - 2p = 100 - 36 = 64$$

$S \leq 10$
 $P \leq \frac{36}{2}$

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$$x \leq t \Rightarrow x = \frac{t}{2}$$

$$g(t) = \frac{bt^2}{c} + 11 \Rightarrow g(x) = \frac{bx^2}{c} + 11$$

$$\frac{-b}{2a} = x_{\min} = \frac{\frac{30}{2}}{\frac{3}{2}} = 10 = V \Rightarrow y_{\min} = \frac{3 \cdot 6}{3} - \frac{6 \cdot 10}{3} + \frac{11 \cdot 9}{3}$$

$C \rightarrow 11$

$$g(x-10) = \frac{b(x-10)^2}{c} + 11 = \frac{bx^2 - 20bx + 100b}{c} + 11 = \frac{bx^2}{c} - \frac{20b}{c}x + \frac{100b}{c} + 11$$

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$$\left. \begin{aligned} i f(x \leq 6) &\Rightarrow f(6) + \underbrace{2f(6) - 2g(6)}_{10} = 16 \Rightarrow f(6) \leq 16 \\ i f(x \leq 0) &\Rightarrow f(0) + \underbrace{2f(0) - 2g(0)}_{10} \leq 16 \Rightarrow f(0) \leq 16 \end{aligned} \right\} \Rightarrow f(x) \leq 2x + 6$$

$$2g(x) \leq 4x + 12 - 16 \Rightarrow 2g(x) \leq 4x - 4 \Rightarrow g(x) \leq 2x - 2$$

$$f \circ g(x) = 2(2x-2) + 6 = 4x + 2 \xrightarrow{y \leq 0} -2 \leq 4x \Rightarrow x \leq \frac{-1}{4}$$

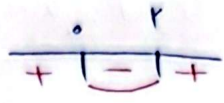
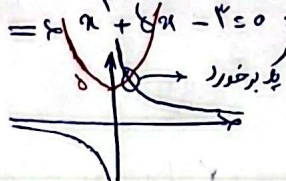
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$$2g(x) - 6 \leq 2x^2 - 4x - 1 \Rightarrow 2g(x) \leq 2x^2 - 4x + 5 \Rightarrow g(x) \leq x^2 - 2x + 2.5$$

$$g \circ f(x) = (2x-6)^2 - 2(2x-6) + 1 = 4x^2 - 24x + 36 - 4x + 12 + 1 = 4x^2 - 28x + 49$$

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در صورتی که
نمی‌توانیم

$f \circ g(x) = g^{-1} \circ f \circ g(x) = g^{-1}(f(g(x))) = g^{-1}(f(x)) = g^{-1}(x^r) = x^{1/r} = x^{1/r}$ $g \circ f(x) = g(f(x)) = g(x^{1/r}) = (x^{1/r})^r = x$ $rx \geq x^r \Leftrightarrow x^r - rx \leq 0$ 	6
$f\left(x - \frac{r}{x}\right) = \left(x - \frac{r}{x}\right)^r = x^r - r + \frac{r}{x^r} + x - \frac{r}{x} = x^r + x - r + \frac{r}{x^r} - \frac{r}{x}$ $\Rightarrow f(x) \leq x^r + x$	7
$f \circ g(x) = g^{-1} \circ f \circ g(x) = g^{-1}(f(g(x))) = g^{-1}(f(x)) = g^{-1}(x^r) = x^{1/r} = x^{1/r}$ $D_f = \{x \in \mathbb{R} : x^r > 0\} = \mathbb{R} \setminus \{0\}$ $D_{f \circ g} = \{x \in D_g : g(x) \in D_f\} = \{x \in \mathbb{R} : x^r > 0\} = \mathbb{R} \setminus \{0\}$ $R_{f \circ g} = \{f(g(x)) : x \in D_{f \circ g}\} = \{f(x) : x \in \mathbb{R} \setminus \{0\}\} = \mathbb{R} \setminus \{0\}$	8
$f(f(x)) = 1 - r f'(x) \Rightarrow f(x) \leq 1 - r x^{r-1}$ $h \circ g(x) = h(g(x)) = h(x^r) = 1 - r(x^r)^{r-1} = 1 - r x^{r^2-r} = 1 - r x^{r(r-1)}$ $x^r - r x^{r-1} + \delta x - r \leq 1 - r x^{r(r-1)} \Rightarrow x^r + \delta x - r \leq 0 \Rightarrow x(x^{r-1} + \delta) \leq r$ $x^{r-1} + \delta \leq \frac{r}{x}$ 	9
$if(x, \epsilon) \rightarrow f(x) \leq r \Rightarrow g(f(x)) = 1 = 4\epsilon x + 1 \Rightarrow \epsilon x \leq 0$	10