

$$D_{g \circ f} = \{x \in D_f \mid f(x) \in D_g\}$$

$$D_f = \{x-1 > 0 \Rightarrow x > 1\} \stackrel{n}{\Rightarrow} D_f = [2, +\infty)$$

$$f_{x-1} > 0 \Rightarrow x-1 > 1 \Rightarrow x > 2$$

$$D_g = \{-x^2 + 6x - 5 > 0 \Rightarrow x^2 - 6x + 5 < 0 \Rightarrow (x-2)(x-4) < 0 \Rightarrow x=2 \Rightarrow D_g = \{2\} \Rightarrow f(x)=2 \Rightarrow f_{x-1} = 2 \Rightarrow x=3 \Rightarrow D_{g \circ f} = \{3\} \quad \checkmark$$

$$f \circ g(x) = f(x^2 - 3x + 1) = 2(x^2 - 3x + 1) - 5 = 2x^2 - 6x + 2 - 5 = 2x^2 - 6x - 3$$

$$g \circ f(x) = g(2x-5) = (2x-5)^2 - 3(2x-5) + 1 = 4x^2 - 20x + 25 - 6x + 15 + 1 = 4x^2 - 26x + 41$$

$$f \circ g(x) = g \circ f(x) \Rightarrow 2x^2 - 6x - 3 = 4x^2 - 26x + 41 \Rightarrow 2x^2 - 20x + 44 = 0 \Rightarrow x^2 - 10x + 22 = 0 \Rightarrow x = \frac{10 \pm \sqrt{100 - 88}}{2} = \frac{10 \pm \sqrt{12}}{2} = 5 \pm \sqrt{3}$$

$$x_1 + x_2 = 10, x_1 x_2 = 22 \Rightarrow x_1 = 5 + \sqrt{3}, x_2 = 5 - \sqrt{3}$$

$$g \circ f(x) = g(f(x)) = g(2x-5) = 2x^2 - 6x - 3$$

$$2x-5 = t \Rightarrow x = \frac{t+5}{2}$$

$$g(t) = 2\left(\frac{t+5}{2}\right)^2 - 6\left(\frac{t+5}{2}\right) - 3 = \frac{2(t^2 + 10t + 25)}{2} - 3(t+5) - 3 = t^2 + 10t + 25 - 3t - 15 - 3 = t^2 + 7t + 7$$

$$g(x-5) = \frac{2(x-5)^2}{2} + 7 = (x-5)^2 + 7$$

$$x_5 = 5 \Rightarrow g_5 = 11 \Rightarrow \min$$

$$f(g(x)) = 2g(x) - 5 = 2(2x^2 - 6x - 3) - 5 = 4x^2 - 12x - 6 - 5 = 4x^2 - 12x - 11$$

$$\Rightarrow g(x) = (x-1)^2$$

$$g \circ f(x) = g(f(x)) = g(2x-5) = (2x-5)^2 = (2x-5)^2$$

$$f(x) - g(x) = -x + 5$$

$$f(x) - g(x) = -x + 5$$

$$\Rightarrow f(x) = 2x + 4$$

$$\Rightarrow g(x) = 2x - 1$$

$$f(g(a)) = 0 \Rightarrow f(2a-1) = 0 \Rightarrow 2(2a-1) + 4 = 0 \Rightarrow 4a - 2 + 4 = 0 \Rightarrow 4a + 2 = 0 \Rightarrow a = -\frac{1}{2}$$

$$\Rightarrow g(a) = -1 \Rightarrow 2a - 1 = -1 \Rightarrow 2a = 0 \Rightarrow a = 0$$

$$f \circ g = f(g(x)) = 9x^2$$

$$g \circ f = g(f(x)) = 9x^2 \Rightarrow x^2(9x) = x^2 \cdot 9x \Rightarrow x(9x) \Rightarrow$$

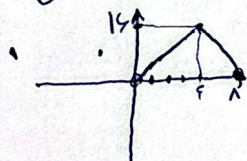
$$\frac{a}{1+|a|} x \in [0, 1] \Rightarrow |a| = 1$$

$$f(x - \frac{1}{x}) = (x - \frac{1}{x})^2 + (x - \frac{1}{x}) \xrightarrow{x - \frac{1}{x} = t} f(t) = t^2 + t$$

$$\Rightarrow f(x) = x^2 + x$$

$$D_{f \circ g} = \{x \in D_g \mid g(x) \in D_f\} \quad D_f \Rightarrow x > 0 \quad 1x - x^2 > 0 \Rightarrow x(1-x) > 0 \Rightarrow \frac{1}{-1+1} =$$

$$D_{f \circ g} = (0, 1) = A$$



$$f \circ g(x) = f(1x - x^2) = 1x - x^2 = R_{f \circ g}$$

$$1x - x^2 \in [0, 1]$$

$$1x - x^2 \in R_{f \circ g} \Rightarrow$$

$$R_{f \circ g} \in [0, 1] \Rightarrow R_{f \circ g} \in [0, 1] \Rightarrow [0, 1] = B$$

$$A \cap B = \{x \mid 0 < x < 1\} \Rightarrow \text{مجموعه}$$

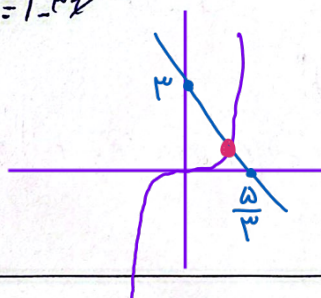
$$h(x-1) = (x-1)^3 + 2(x-1) + 1 = x^3 - 3x^2 + 3x - 1 + 2x - 2 + 1 = x^3 - 3x^2 + 5x - 2$$

$$f(h(x)) = 1 - 3h^2(x) \Rightarrow f(t) = 1 - 3t^2 \Rightarrow f(x) = 1 - 3x^2$$

$$x^3 - 3x^2 + 5x - 2 = 1 - 3x^2 \Rightarrow x^3 + 5x - 3 = 0$$

$$x^3 = -5x + 3$$

یک بر خود دارند $\rightarrow$ یک ریشه



$$g(f(x)) = ax^2 + 2x$$

$$g(1) = 1$$

$$\Rightarrow f(x) = 1 = x - \frac{x}{x} = 1 \Rightarrow x^2 - x = 1 \Rightarrow x^2 - x - 1 = 0$$

$$(x-1)(x+1) = 0 \Rightarrow \begin{cases} x=1 \Rightarrow g(f(1)) = g(1) = 4a+1=1 \Rightarrow a=0 \\ x=-1 \Rightarrow g(f(-1)) = g(1) = -a-1=1 \Rightarrow a=-2 \end{cases}$$