

۲۰ آتین

الف) $x^2 - 5x - 4 = 0$ $x^2 - 5x - 4 = 0$ $(x-9)(x+4) = 0$ $x = 9, -4 \rightarrow x = \frac{9}{4}, -\frac{4}{4} \pm x = \frac{9}{4}, -\frac{4}{4}$ ب) $3x^2 - 8x - 3 = 0$ $x^2 - 8x - 9 = 0$ $(x-9)(x+1) = 0$ $x = 9, -1 \rightarrow x = \frac{9}{3}, -\frac{1}{3}$	ج) $4x^2 - 4x - 3 = 0$ $x^2 - 4x - 12 = 0$ $(x-4)(x+3) = 0$ $x = 4, -3 \rightarrow x = \frac{4}{4}, -\frac{3}{4} \rightarrow x = \frac{4}{4}, -\frac{3}{4}$ د) $2x^2 - 5x - 12 = 0$ $x^2 - 5x - 12 = 0$ $(x-8)(x+3) = 0$ $x = 8, -3 \rightarrow x = \frac{8}{2}, -\frac{3}{2} \rightarrow x = \frac{8}{2}, -\frac{3}{2}$	۱
الف) $x^2 - 5x - 12 = 0$ $(x-7)(x+2) = 0$ $x = 7, -2$ ب) $2x^2 + 5x + 3 = 0$ $x^2 + 5x + 4 = 0$ $(x+2)(x+3) = 0$ $x = -2, -3 \rightarrow x = -\frac{2}{1}, -\frac{3}{1}$	ج) $x^2 + 2x + 3 = 0$ $\Delta = b^2 - 4ac$ $\Delta = (2)^2 - 4(1)(3) = -8$ $\Delta < 0$ خط مشی ندارد د) $3x^2 + 3x - 2 = 0$ $\Delta = b^2 - 4ac$ $\Delta = (3)^2 - 4(-2)(3) = 21$ $x = \frac{-b \pm \sqrt{\Delta}}{2a} \rightarrow x = \frac{-3 \pm \sqrt{21}}{6} \rightarrow x = \frac{-3 + \sqrt{21}}{6}, x = \frac{-3 - \sqrt{21}}{6}$	۲
$A = \{1, 2, 3\} \rightarrow$ عضو $B = \{2, 3\} \rightarrow$ عضو $A \times B - B^c = ?$ $\{(1, 2), (1, 3), (2, 2), (2, 3), (3, 2), (3, 3)\} - \{(2, 2), (2, 3), (3, 2), (3, 3)\} = \{(1, 2), (1, 3)\}$ $n(A \times B) - n(B^c) = n(A) \times n(B) - n(B) \times n(B) \rightarrow (3 \times 2) - (2 \times 2) = 2$		۳
$f(m) = \{(3, m^2), (2, 1), (-3, m), (-2, m), (3, m+2), (m, 4)\}$ $(3, m^2) = (3, m+2) \rightarrow m^2 = m+2 \rightarrow m^2 - m - 2 = 0 \rightarrow a+c=b \rightarrow m = -1, m = 2$ $g(m) = \{(3, m^2), (2, 1), (m, 4), (3, m+2), (-2, m), (-1, 3)\}$ $(3, m^2) = (3, m+2) \rightarrow m^2 = m+2 \rightarrow m^2 - m - 2 = 0 \rightarrow a+c=b \rightarrow m = -1, m = 2$	$m = -1 \rightarrow (3, 4) \neq (2, 1) \times$ $m = 2 \rightarrow (3, 8) \neq (2, 1) \times$ $m = -1 \rightarrow (-1, 4) \neq (-1, 3) \times$ $m = 2 \rightarrow (2, 1) \neq (2, 4) \times$ توجه: نسبت به ازای هیچ مقولای از	۴
$f(m) = \{(1, a^2), (1, 3a-2), (a^2, 3a+2)\}$ $(1, a^2) = (1, 3a-2) \rightarrow a^2 = 3a-2 \rightarrow a^2 - 3a + 2 = 0 \rightarrow a+b+c \rightarrow a = 2, a = 1$	$a = 1 \rightarrow (1, 1) \neq (1, 1) \times$ $a = 2 \rightarrow (1, 4) \neq (4, 8) \times$ بازای $a=1$ به ازای $a=2$ به ازای $a=1$ به ازای $a=2$	۵

$f(x) = \begin{cases} x^2 - 1 & x < 1 \\ ax + b & 1 \leq x \leq 2 \\ x^3 & x > 2 \end{cases}$	① $x=1 \rightarrow x^2 - 1 = a(1) + b$ ② $x=2 \rightarrow a(2) + b = 2^3$ ① $(1)^2 - 1 = a(1) + b$ $0 = a + b$ ② $a(2) + b = (2)^3$ $2a + b = 8$	①, ② $\begin{cases} (a+b=0) \times -1 \\ 2a+b=8 \end{cases}$ $\begin{cases} a+b=0 \\ 2a+b=8 \end{cases}$ $a+b - a - b = 8 - 0$ $-a = 8$ $a = -8$ $a+b=0$ $-8+b=0$ $b=8$ $a \times b = -8 \times 8 = -64$	9
$f(x) = \begin{cases} 2x - 3 & x > 4 \\ 4 - 3x & x < 4 \end{cases}$	$x=4 \rightarrow 2x - 3 = 4 - 3x$ $2(4) - 3 = 4 - 3(4)$ $8 - 3 = 4 - 12$ $5 = -8$ $5 = -8$		7
الف) $y^3 - 4y = x^2 - 4x + 3$ X $x=3 \rightarrow y^3 - 4y = 3^2 - 4(3) + 3$ مثال نقض ✓ $x=4 \rightarrow y^3 - 4y = 4^2 - 4(4) + 3$ $y^3 - 4y = 0$ $y(y^2 - 4) = 0 \rightarrow y=0$ یا $y = \pm 2$ ب) $x + y = 3$ X $x=1 \rightarrow 1 + y = 3$ مثال نقض ✓ $y = 2$ $y = \pm 2$	ب) $y = \sqrt{1 - \cos x}$ X $1 - \cos x > 0 \Rightarrow 1 < \cos x$ همیشه برقرار است $x=0 \rightarrow y = \sqrt{1 - 1}$ $y = 0 \rightarrow y = 0$ ج) $y^2 + x^2 + 2x + 1 = 0$ $y^2 + (x+1)^2 = 0 \Rightarrow y=0$ و $x=-1$	مثال نقض $x=4$ به ازای y مقدار مشخصی ندارد مثال نقض جمع 2 مقدار + صفر است	1
الف) $\sqrt[3]{y} + x^2 = 2x - 2$ ✓ $x_1 = x_2 \Rightarrow y_1 = y_2$ $\sqrt[3]{y_1} = -x_1^2 + 2x_1 - 2$ $\sqrt[3]{y_2} = -x_2^2 + 2x_2 - 2$ $\sqrt[3]{y_1} = \sqrt[3]{y_2}$ چون $\frac{y_1}{y_2} = 1$ $y_1 = y_2$ ✓	ب) $y^3 + 2y = 5x$ ✓ $x_1 = x_2 \Rightarrow y_1 = y_2$ $y_1^3 + 2y_1 = 5x_1$ $y_2^3 + 2y_2 = 5x_2$ $y_1^3 + 2y_1 = y_2^3 + 2y_2$ $y_1^3 - y_2^3 + 2y_1 - 2y_2 = 0$ $(y_1 - y_2)(y_1^2 + y_1y_2 + y_2^2) + 2(y_1 - y_2) = 0$	$y_1 = y_2$ $(y_1 - y_2)(y_1^2 + y_1y_2 + y_2^2) + 2(y_1 - y_2) = 0$ $y_1 = y_2$	40
الف) $x \sin y = y \sin x$ X $x=\pi \rightarrow \pi \sin y = y \sin(\pi)$ $\pi \sin y = 0 \Rightarrow y = \pi$ مثال نقض ✓ به ازای $x=\pi$ مقدارهای مختلفی برای y خواهیم داشت به صورت $y = k\pi$	ب) $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 2$ ✓ $\frac{x}{y} + \frac{y}{x} + 2\sqrt{\frac{xy}{xy}} = 4$ $\frac{x}{y} + \frac{y}{x} = 2$ $\frac{x^2 + y^2}{xy} = 2$ $x^2 + y^2 = 2xy \rightarrow x^2 - 2xy + y^2 = 0 \Rightarrow (x-y)^2 = 0 \rightarrow x-y=0$ $x=y$	مثال نقض	10