

۲. آزمون ضریب همل نوشتن!

الف) $4x^2 - 5x - 4 = 0 \rightarrow x^2 - 5x - 34 = 0$

$(x+8)(x-9) = 0$

$x = -\frac{8}{1} = -8$ $x = \frac{9}{1} = 9$

$x = \frac{13}{2} \pm \frac{1}{2}$

ب) $3x^2 - 8x - 4 = 0 \rightarrow x^2 - 8x - 9 = 0$

$(x+1)(x-9) = 0$

$x = -\frac{1}{1} = -1$ $x = \frac{9}{1} = 9$

$x = -\frac{1}{3} \pm 3$

ج) $8x^2 - 5x - 3 = 0 \rightarrow x^2 - 5x - 12 = 0$

$(x-4)(x+3) = 0$

$x = \frac{4}{1} = 4$ $x = -\frac{3}{1} = -3$

$x = -\frac{1}{2} \pm \frac{3}{2}$

د) $2x^2 - 5x - 12 = 0 \rightarrow x^2 - 5x - 12 = 0$

$(x+3)(x-8) = 0$

$x = -\frac{3}{1} = -3$ $x = \frac{8}{1} = 8$

$x = 5 \pm \frac{13}{2}$

ه) $x^2 - 5x - 12 = 0 \rightarrow (x+2)(x-7) = 0$

$x = -2$ $x = 7$

و) $2x^2 + 5x + 3 = 0 \rightarrow 1 = b^2 - 4ac \rightarrow \Delta = 25 - 12 = 13$

$x = \frac{-b \pm \sqrt{\Delta}}{2a} \rightarrow x = \frac{-5 \pm \sqrt{13}}{4}$

$x = -1 \pm \frac{3}{2}$

$x = \frac{-5 - 1}{2} = -3$ $x = \frac{-5 + 1}{2} = -2$

ز) $x^2 + 2x + 3 = 0 \rightarrow \Delta = b^2 - 4ac = 4 - 12 = -8$ $\Delta < 0$ $\rightarrow$ جواب ندارد

ح) $3x^2 + 4x - 2 = 0 \rightarrow \Delta = 16 - 4(3)(-2) = 16 + 24 = 40$

$x = \frac{-b \pm \sqrt{\Delta}}{2a} \rightarrow x = \frac{-4 \pm \sqrt{40}}{6}$

$x = \frac{-4 + \sqrt{40}}{6}$ $x = \frac{-4 - \sqrt{40}}{6}$

$$A \times B = \{(1, 2), (1, 3), (2, 2), (2, 3), (3, 2), (3, 3)\}$$

$$B = \{2, 3\}$$

$$B^x = B \times B = \{(2, 2), (2, 3), (3, 2), (3, 3)\}$$

$$A = \{1, 2, 3\}$$

$$A \times B - B^x = \{(1, 2), (1, 3)\}$$

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$$\text{الف) } f(x) = \{(w, m^x), (x, 1), (-x, m), (-x, m), (w, m+x), (m, x)\}$$

$$m^x = m + x$$

$$m^x - m - x = 0$$

$$(m - x)(m + 1) = 0$$

$$m = x \quad \times$$

$$m = -1 \quad \checkmark$$

$$(x, 1) \rightarrow (m, x) \rightarrow (-1, x)$$

$$(x, 1) \rightarrow (m, x) \rightarrow (x, x)$$

$$\text{ب) } g(x) = \{(w, m^x), (x, 1), (m, x), (w, m+x), (-x, m), (-1, x)\}$$

$$m^x = m + x$$

$$m^x - m - x = 0$$

$$(m - x)(m + 1) = 0$$

$$m = x \quad \times$$

$$m = -1 \quad \times$$

$$(x, 1)$$

$$(-1, x)$$

$$(m, x) \rightarrow (x, x)$$

$$(m, x) \rightarrow (-1, x)$$

م به ازای هیچ مقداری تابع نیست

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$$f(x) = \{(1, a), (1, 3a-x), (a, 3a+x)\}$$

$$a^x = 3a - x$$

$$a^x - 3a + x = 0$$

$$(a - 1)(a - x) = 0$$

م به ازای یک مقدار تابع است رابطه (a = x)

$$a = 1 \quad \times$$

$$a = x \quad \checkmark$$

$$f(x) = \{(1, x), (1, x), (x, x)\} \Rightarrow f(x) = \{(1, x), (x, x)\}$$

$$\rightarrow f(x) = \{(1, 1), (1, 1), (1, x)\} \Rightarrow f(x) = \{(1, 1), (1, x)\}$$

$$f(x) = \begin{cases} x^2 - 1 & x \leq 1 \\ ax + b & 1 \leq x \leq 2 \\ x & x > 2 \end{cases}$$

$$(x^2 - 1) = (ax + b) \xrightarrow{x=1} 1 - 1 = a + b \rightarrow a + b = 0 \rightarrow a = -b$$

$$x^2 = ax + b \xrightarrow{x=2} 4 = 2a + b$$

$$\begin{cases} a + b = 0 \\ 2a + b = 4 \end{cases} \rightarrow \begin{cases} -a - b = 0 \\ 2a + b = 4 \end{cases}$$

$$\begin{aligned} &\underline{a = 1} \\ &\underline{b = -1} \end{aligned} \Rightarrow a \times b = 1 \times (-1) = -1$$

$$f(x) = \begin{cases} x^2 - 3 & x > k \\ \varepsilon - x^2 & x \leq k \end{cases}$$

$$x^2 - 3 = \varepsilon - x^2 \xrightarrow{x=k} k^2 - 3 = \varepsilon - k^2 \rightarrow 2k^2 = \varepsilon + 3$$

$$k = \frac{\sqrt{\varepsilon + 3}}{2}$$

$$\text{a) } y'' - 4y = x^2 - \varepsilon x + 3 \rightarrow \text{X p.e.}$$

$$y'' - 4y = \text{X p.e.}$$

$$\text{b) } |x| + |y| = 3 \rightarrow x = 0 \rightarrow |y| = 3 \rightarrow y = \pm 3$$

$$\text{X p.e.}$$

$$\text{c) } |y| = \sqrt{1 - \cos x} \rightarrow -1 \leq \cos x \leq 1 \rightarrow \cos x = -1 \rightarrow \sqrt{1 - (-1)} = \sqrt{2} \rightarrow |y| = \sqrt{2} \rightarrow y = \pm \sqrt{2}$$

$$\text{X p.e.}$$

$$\text{d) } y^2 + \underbrace{x^2 + 2x + 1}_{(x+1)^2} = 0 \rightarrow y^2 + (x+1)^2 = 0 \Rightarrow \begin{cases} y = 0 \\ x + 1 = 0 \rightarrow x = -1 \end{cases}$$

$$\sqrt{0} \quad \downarrow \quad (-1, 0) \checkmark$$

الف) $\sqrt[3]{y} + x^2 = 2x - 2$

$$\begin{cases} \sqrt[3]{y_1} = -x^2 + 2x - 2 \\ \sqrt[3]{y_2} = -x^2 + 2x - 2 \end{cases}$$

$$\rightarrow -x^2 + 2x = -x^2 + 2x \rightarrow -x^2 + 2x - 2 = -x^2 + 2x - 2$$

$$\rightarrow \sqrt[3]{y_1} = \sqrt[3]{y_2} \rightarrow (\sqrt[3]{y_1})^3 = (\sqrt[3]{y_2})^3 \rightarrow y_1 = y_2 \quad \checkmark$$

$$\therefore y^3 + 2y = 2x$$

$$\begin{cases} y_1^3 + 2y_1 = 2x \\ y_2^3 + 2y_2 = 2x \end{cases}$$

$$\rightarrow 2x = 2x \rightarrow y_1^3 + 2y_1 = y_2^3 + 2y_2 \rightarrow y_1^3 - y_2^3 + y_1 - y_2 = 0$$

$$\rightarrow (y_1 - y_2)(y_1^2 + y_1 y_2 + y_2^2) + y_1 - y_2 = 0$$

$$\rightarrow (y_1 - y_2)(y_1^2 + y_1 y_2 + y_2^2 + 1) = 0$$

$$y_1 = y_2$$

$$y_1^2 + y_1 y_2 + y_2^2 + 1 = 0$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$y_1 = \frac{-y_2 \pm \sqrt{y_2^2 - 4(y_2^2 + 1)}}{2} \rightarrow -y_2^2 - 4 \rightarrow \Delta < 0$$

پس تابع است ✓

$$\text{الف) } x \sin y = y \sin x \rightarrow x = \pi \rightarrow \pi \sin y = y \sin \pi$$

$$\rightarrow \pi \sin y = 0$$

$$\rightarrow \sin y = 0 \quad \textcircled{\phi}$$

$$y = n\pi \rightarrow n = \pi \text{ یعنی برای } n = \pi$$

صفرین مقدار y وجود دارد

~~تابع~~

$$\text{ب) } \sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 2$$

$$\sqrt{\frac{x}{y}} = t \Rightarrow t + \frac{1}{t} = 2 \rightarrow t^2 + 1 = 2t$$

$$t^2 - 2t + 1 = 0$$

$$(t-1)^2 = 0$$

$$t = 1$$

$$\sqrt{\frac{x}{y}} = 1$$

$$\frac{x}{y} = 1 \rightarrow x = y$$

تابع ✓

$$\text{ب-۱۰) } \sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 2$$

این عبارت تنها زمانی برقرار است که $\sqrt{\frac{x}{y}} = 1$ باشد پس

$$\sqrt{\frac{x}{y}} = 1 \Rightarrow \frac{x}{y} = 1 \Rightarrow x = y \text{ و } (x, y \neq 0)$$

