

سوال ۱

الف) $4x^2 - 5x - 4 = 0$
 $x^2 - 5x - 4 = 0 \Rightarrow (x-9)(x+1) = 0$
 $x_1 = \frac{9}{4} = \frac{9}{4}$, $x_2 = \frac{-5}{4} = -\frac{5}{4}$

ب) $3x^2 - 11x - 9 = 0$
 $x^2 - 11x - 9 = 0 \Rightarrow (x-9)(x+1) = 0$
 $x_1 = \frac{9}{3} = 3$, $x_2 = \frac{-1}{3} = -\frac{1}{3}$

ج) $5x^2 - 8x - 4 = 0$
 $x^2 - 8x - 12 = 0 \Rightarrow (x-6)(x+2) = 0$
 $x_1 = \frac{4}{2} = 2$, $x_2 = \frac{-2}{2} = -1$

د) $2x^2 - 5x - 12 = 0$
 $x^2 - 5x - 12 = 0 \Rightarrow (x-1)(x+3) = 0$
 $\Rightarrow x = \frac{1}{2} = \frac{1}{2}$, $x_2 = \frac{-3}{2} = -\frac{3}{2}$

سوال ۲

الف) $x^2 - 5x - 12 = 0$
 $(x-7)(x+2) = 0 \Rightarrow x_1 = 7$, $x_2 = -2$

ب) $2x^2 + 5x + 3 = 0$
 $a+c = b \Rightarrow x_1 = -1$, $x_2 = \frac{-3}{2} = -\frac{3}{2}$

ج) $x^2 + 2x + 3 = 0$
 $\Delta = 2 - 12 = -10 \Rightarrow \Delta < 0 \Rightarrow$ معادله ریشه ندارد.

د) $3x^2 + 3x - 2 = 0$
 $\Delta = 9 + 24 = 33 \Rightarrow x_1 = \frac{-3 + \sqrt{33}}{6}$, $x_2 = \frac{-3 - \sqrt{33}}{6}$

سوال ۳

$B = \{2, 3\}$, $A = \{1, 2, 3\}$, $A \times B - B^2 = ?$
 $A \times B = \{(1, 2), (1, 3), (2, 2), (2, 3), (3, 2), (3, 3)\}$
 $B \times B = \{(2, 2), (2, 3), (3, 2), (3, 3)\}$
 $\Rightarrow A \times B - B^2 = \{(1, 2), (1, 3)\} \Rightarrow \bar{C}$

سوال ۴

الف) $f(x) = \{(3, m^2), (4, 1), (-3, m), (-2, m), (3, m+2), (m, 4)\}$
 $m^2 = m + 2 \Rightarrow m^2 - m - 2 = 0 \Rightarrow \Delta = 9 \Rightarrow m_1 = \frac{1+3}{2} = 2$, $m_2 = \frac{1-3}{2} = -1$

ب) $g(x) = \{(3, m^2), (4, 1), (m, 4), (3, m+2), (-2, m), (-1, 3)\}$
 $m^2 = m + 2 \Rightarrow m^2 - m - 2 = 0 \Rightarrow m_1 = 2$, $m_2 = -1$

سوال ۵

$f(x) = \{(1, a^2), (1, 3a-2), (a^2, 3a+2)\}$
 $a^2 - 3a + 2 = 0 \Rightarrow a^2 - 3a + 2 = 0 \Rightarrow a = 1$, $a = 2$

سوال ۶

$f(x) = \begin{cases} x^2 - 1 & x \leq 1 \\ ax + b & 1 < x < 2 \\ x^3 & x \geq 2 \end{cases}$
 $n=1 \Rightarrow a+b=0$
 $x=2 \Rightarrow 2a+b=1$
 $\Rightarrow \begin{cases} -a-b=0 \\ 2a+b=1 \end{cases} \Rightarrow a=1 \Rightarrow b=-1$
 $a \times b = 1 \times (-1) = -1$

سوال ۷

$f(x) = \begin{cases} 2x-3 & x \geq k \\ 4-3x & x \leq k \end{cases}$
 $n=k \Rightarrow 2k-3 = 4-3k \Rightarrow 5k = 7 \Rightarrow k = \frac{7}{5}$

سوال ۸

الف) $4y^2 - 4y = x^2 - 2x + 3$
 $y^2 - y = \frac{x^2 - 2x + 3}{4}$

ب) $|x| + |y| = 3$
 $x=1 \Rightarrow |y| = 2 \Rightarrow y = \pm 2$

ج) $|y| = \sqrt{1 - \cos x}$
 $1 - \cos x = 0 \Rightarrow \cos x = 1 \Rightarrow x = 0$

د) $y^2 + x^2 + 2x + 1 = 0 \Rightarrow y^2 + (x+1)^2 = 0 \Rightarrow x = -1$

ه) $(y_1 - y_2)(y_1^2 + y_1 y_2 + y_2^2) = 0$
 $y_1 = y_2$
 $\Delta = y^2 - 2(y^2 - 4) = y^2 - 2y^2 + 8 = -y^2 + 8 \Rightarrow \Delta > 0$

سوال ۹

الف) $\sqrt[3]{y} + x^2 = 2x - 2$

$\sqrt[3]{y_1} = \sqrt[3]{y_2}$

$$\begin{cases} \text{① } \sqrt[3]{y_1} = -x^2 + 2x - 2 \\ \text{② } \sqrt[3]{y_2} = -x^2 + 2x - 2 \end{cases} \quad \text{①=②}$$

$y_1 = y_2 \Rightarrow$ جمع است

ب) $y^3 + 2y = 2x$

$$\begin{cases} y_1^3 + 2y_1 = 2x \\ y_2^3 + 2y_2 = 2x \end{cases}$$

$y_1^3 + 2y_1 = y_2^3 + 2y_2$

$y_1^3 - y_2^3 = 2y_2 - 2y_1$

$(y_1 - y_2)(y_1^2 + y_1 y_2 + y_2^2) = -2(y_1 - y_2)$

$\Rightarrow (y_1 - y_2)(y_1^2 + y_1 y_2 + y_2^2 + 2) = 0$

$y_1 - y_2 = 0 \Rightarrow y_1 = y_2$

$\Rightarrow y_1^2 + y_1 y_2 + y_2^2 + 2 = 0$

$\Rightarrow y_1 = \frac{-y_2 \pm \sqrt{\Delta}}{2} \Rightarrow \Delta = y_2^2 - 4(y_2^2 + 2)$

$\Rightarrow \Delta = y_2^2 - 4y_2^2 - 8 = -3y_2^2 - 8 \Rightarrow \Delta < 0 \Rightarrow$ رابطه معادله را ندارد $\Rightarrow y_1 = y_2 \Rightarrow$ جمع است

سوال ۱۰

الف) $n \sin y = y \sin n$

$n=0 \Rightarrow 0 = y \sin 0 \Rightarrow$ جمع است $y = 0$

ب) $\sqrt{\frac{n}{y}} + \sqrt{\frac{y}{n}} = 2$

$a \times b = \sqrt{\frac{n}{y} \cdot \frac{y}{n}} = 1 \Rightarrow \begin{cases} a+b=2 \\ a \cdot b=1 \end{cases} \Rightarrow a, b=1$

$\Rightarrow \sqrt{\frac{n}{y}} = 1 \Rightarrow \frac{n}{y} = 1 \Rightarrow n = y \Rightarrow$ جمع است