

کتاب 1

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الف) $4x^2 - 5x - 4 = 0$

$$x^2 - 5x - 4 = 0 \Rightarrow (x-9)(x+1) = 0$$

$$x_1 = \frac{9}{4} = \frac{9}{4}, x_2 = \frac{-5}{4} = -\frac{5}{4}$$

ب) $3x^2 - 11x - 9 = 0$

$$x^2 - 11x - 9 = 0 \Rightarrow (x-9)(x+1) = 0$$

$$x_1 = \frac{9}{3} = 3, x_2 = \frac{-1}{3}$$

ج) $5x^2 - 8x - 4 = 0$

$$x^2 - 8x - 12 = 0 \Rightarrow (x-6)(x+2) = 0$$

$$x_1 = \frac{8}{5} = \frac{8}{5}, x_2 = \frac{-4}{5} = -\frac{4}{5}$$

د) $2x^2 - 5x - 12 = 0$

$$x^2 - 5x - 12 = 0 \Rightarrow (x-1)(x+3) = 0$$

$$\Rightarrow x = \frac{1}{2} = \frac{1}{2}, x_2 = \frac{-3}{2}$$

سوال 1

الف) $x^2 - 2x - 12 = 0$

$$(x-4)(x+3) = 0 \Rightarrow x_1 = 4, x_2 = -3$$

ب) $\frac{2}{a}x^2 + \frac{5}{b}x + \frac{3}{c} = 0$

$$a+c = b \Rightarrow x_1 = -1, x_2 = \frac{c}{a} = -\frac{3}{2}$$

ج) $x^2 + 2x + 3 = 0$

$$\Delta = 2^2 - 12 = -8 \Rightarrow \Delta < 0 \Rightarrow \text{معادله حقیقی ندارد}$$

د) $3x^2 + 3x - 2 = 0$

$$\Delta = 9 + 24 = 33 \Rightarrow x_1 = \frac{-3 + \sqrt{33}}{6}, x_2 = \frac{-3 - \sqrt{33}}{6}$$

سوال 2

$B = \{2, 3\}, A = \{1, 2, 3\}, A \times B - B^2 = ?$

$$A \times B = \{(1, 2), (1, 3), (2, 2), (2, 3), (3, 2), (3, 3)\}$$

$B \times B = \{(2, 2), (2, 3), (3, 2), (3, 3)\}$

$$\Rightarrow A \times B - B^2 = \{(1, 2), (1, 3), (2, 2), (3, 2), (3, 3)\} \Rightarrow 5$$

سوال 3

الف) $f(x) = \{(3, m^2), (4, 1), (-3, m), (-2, m), (3, m+2), (m, 4)\}$

$$m^2 = m + 2 \Rightarrow m^2 - m - 2 = 0 \Rightarrow \Delta = 9 \Rightarrow m_1 = \frac{1+3}{2} = 2, m_2 = \frac{1-3}{2} = -1$$

ب) $g(x) = \{(3, m^2), (4, 1), (m, 4), (3, m+2), (-2, m), (-1, 3)\}$

$$m^2 = m + 2 \Rightarrow m^2 - m - 2 = 0 \Rightarrow m_1 = 2, m_2 = -1$$

سوال 4

$f(x) = \{(1, a^2), (1, 3a-2), (a^2, 3a+2)\}$ $\Rightarrow a^2 - 3a + 2 = 0 \Rightarrow a + b + c = 0 \Rightarrow a = 1, a = 2$

سوال 5

$$f(x) = \begin{cases} x^2 - 1 & x \leq 1 \\ ax + b & 1 < x < 2 \\ x^3 & x \geq 2 \end{cases}$$

$$x=1 \Rightarrow a+b=0$$

$$x=2 \Rightarrow 2a+b=1$$

$$\Rightarrow \begin{cases} -a-b=0 \\ 2a+b=1 \end{cases}$$

$$a=1 \Rightarrow b=-1$$

$$a \times b = 1 \times (-1) = -1$$

سوال 6

$$f(x) = \begin{cases} 2x-3 & x \geq k \\ 4-3x & x \leq k \end{cases}$$

$$\xrightarrow{x=k} 2k-3 = 4-3k \Rightarrow 5k=7 \Rightarrow k = \frac{7}{5}$$

سوال 7

الف) $4y^2 - 4y = x^2 - 2x + 3$

تبدیل

ب) $|x| + |y| = 3$

$$\xrightarrow{x=1} |y| = 2 \Rightarrow y = \pm 2 \Rightarrow \text{تبدیل}$$

ج) $|y| = \sqrt{1-\cos x} \Rightarrow 1-\cos x \geq 0 \Rightarrow 1 \geq \cos x \Rightarrow \cos x = 1 \Rightarrow y = 1 \Rightarrow y = \pm 1 \Rightarrow \text{تبدیل}$

د) $y^2 + x^2 + 2x + 1 = 0 \Rightarrow y^2 + (x+1)^2 = 0 \Rightarrow |y| = 0 \Rightarrow \text{تبدیل}$

$$\Rightarrow (y_1 - y_2)(y_1^2 + y_1 y_2 + y_2^2 - 4) = 0$$

$$y_1 = y_2$$

$$\Delta = y_1^2 - 2(y_1^2 - 4) = y_1^2 - 2y_1^2 + 8 = -y_1^2 + 8 \Rightarrow \Delta \geq 0$$

تبدیل

سوال ۹

الف) $\sqrt[3]{y} + x^2 = 2x - 2$

$\sqrt[3]{y_1} = \sqrt[3]{y_2}$

$y_1 = y_2 \Rightarrow$ جمع است

$$\begin{cases} \textcircled{1} \sqrt[3]{y_1} = -x^2 + 2x - 2 \\ \textcircled{2} \sqrt[3]{y_2} = -x^2 + 2x - 2 \end{cases} \quad \textcircled{1} = \textcircled{2}$$

ب) $y^3 + 2y = 2x$

$$\begin{cases} y_1^3 + 2y_1 = 2x \\ y_2^3 + 2y_2 = 2x \end{cases}$$

$y_1^3 + 2y_1 = y_2^3 + 2y_2$

$y_1^3 - y_2^3 = 2y_2 - 2y_1$

$(y_1 - y_2)(y_1^2 + y_1 y_2 + y_2^2) = -2(y_1 - y_2)$

$\Rightarrow (y_1 - y_2)(y_1^2 + y_1 y_2 + y_2^2 + 2) = 0$

$y_1 - y_2 = 0 \Rightarrow y_1 = y_2$

$\Rightarrow y_1^2 + y_1 y_2 + y_2^2 + 2 = 0$

$\Rightarrow y_1 = \frac{-y_2 \pm \sqrt{\Delta}}{2} \Rightarrow \Delta = y_2^2 - 4(y_2^2 + 2)$

$\Rightarrow \Delta = y_2^2 - 4y_2^2 - 8 = -3y_2^2 - 8 \Rightarrow \Delta < 0 \Rightarrow$ رابطه ی مربعی ندارد $\Rightarrow y_1 = y_2 \Rightarrow$ جمع است

سوال ۱۰

الف) $n \sin y = y \sin n$

$n=0$

$0 = y \sin 0 \Rightarrow$ جمع است $y = 0$ جواب

ب) $\sqrt{\frac{n}{y}} + \sqrt{\frac{y}{n}} = 2$

$a \times b = \sqrt{\frac{n}{y} \cdot \frac{y}{n}} = 1$

$$\begin{cases} a+b=2 \\ a \cdot b=1 \end{cases} \Rightarrow a, b=1$$

$\Rightarrow \sqrt{\frac{n}{y}} = 1 \Rightarrow \frac{n}{y} = 1 \Rightarrow n = y \Rightarrow$ جمع است