

۱۹.۵ آزمون ضرب نقیسه

الف) $y = \frac{x+3}{x^3 + 3x^2 - 10x + 3} = \frac{x+3}{(x-1)(x^2 + 4x - 3)} = \frac{x+3}{(x-1)(x+6)}$

$Df = \mathbb{R} - \{-3, -1\}$

Partial fractions: $\frac{x+3}{(x-1)(x+6)} = \frac{A}{x-1} + \frac{B}{x+6}$
 $\frac{x+3}{(x-1)(x+6)} = \frac{A(x+6) + B(x-1)}{(x-1)(x+6)}$
 $x+3 = A(x+6) + B(x-1)$
 $x+3 = Ax + 6A + Bx - B$
 $x+3 = (A+B)x + (6A-B)$
 $\begin{cases} A+B=1 \\ 6A-B=3 \end{cases} \Rightarrow \begin{matrix} A = \frac{4}{7} \\ B = \frac{3}{7} \end{matrix}$

ب) $y = \frac{x+3}{x^3 + 9x^2 + 10x + 3} = \frac{x+3}{(x+1)(x^2 + 8x + 3)}$

$Df = \mathbb{R} - \{-3, -1, -\frac{1}{8}\}$

Partial fractions: $\frac{x+3}{(x+1)(x^2 + 8x + 3)} = \frac{A}{x+1} + \frac{Bx+C}{x^2 + 8x + 3}$
 $\frac{x+3}{(x+1)(x^2 + 8x + 3)} = \frac{A(x^2 + 8x + 3) + (Bx+C)(x+1)}{(x+1)(x^2 + 8x + 3)}$
 $x+3 = A(x^2 + 8x + 3) + (Bx+C)(x+1)$
 $x+3 = Ax^2 + 8Ax + 3A + Bx^2 + Bx + Cx + C$
 $x+3 = (A+B)x^2 + (8A+B+C)x + (3A+C)$
 $\begin{cases} A+B=0 \\ 8A+B+C=1 \\ 3A+C=3 \end{cases} \Rightarrow \begin{matrix} A = -\frac{1}{7} \\ B = \frac{1}{7} \\ C = \frac{10}{7} \end{matrix}$

الف) $y = \frac{x+3}{x^3 - 2x^2 + 4x - 1} \Rightarrow Df = \mathbb{R} - \{1\}$

Partial fractions: $\frac{x+3}{x^3 - 2x^2 + 4x - 1} = \frac{A}{x-1} + \frac{Bx+C}{x^2 + 3x - 1}$
 $\frac{x+3}{x^3 - 2x^2 + 4x - 1} = \frac{A(x^2 + 3x - 1) + (Bx+C)(x-1)}{(x-1)(x^2 + 3x - 1)}$
 $x+3 = A(x^2 + 3x - 1) + (Bx+C)(x-1)$
 $x+3 = Ax^2 + 3Ax - A + Bx^2 - Bx + Cx - C$
 $x+3 = (A+B)x^2 + (3A-B+C)x + (-A-C)$
 $\begin{cases} A+B=0 \\ 3A-B+C=1 \\ -A-C=3 \end{cases} \Rightarrow \begin{matrix} A = \frac{1}{4} \\ B = -\frac{1}{4} \\ C = -\frac{3}{4} \end{matrix}$

ب) $y = \sqrt{\frac{x+3}{x^3 - 2x^2 + 4x - 1}}$

$Df = (-\infty, -3] \cup (1, +\infty)$

Graph of $y = \sqrt{\frac{x+3}{x^3 - 2x^2 + 4x - 1}}$ showing domain restrictions at $x = -3$ and $x = 1$.

$y = \frac{2}{x^2 - 5|x-1| - 2x + 5}$

Case 1: $x \geq 1 \Rightarrow y = \frac{2}{x^2 - 5(x-1) - 2x + 5} = \frac{2}{x^2 - 7x + 10} = \frac{2}{(x-2)(x-5)}$

Case 2: $x \leq 1 \Rightarrow y = \frac{2}{x^2 + 5x - 5 - 2x + 5} = \frac{2}{x^2 + 3x} = \frac{2}{x(x+3)}$

$Df = \mathbb{R} - \{-3, 0, 2, 5\}$

الف) $y = \sqrt{[x] - 2} \quad [x] \geq 2 \Rightarrow Df = (-\infty, -2]$

ب) $y = \frac{5x^2 + 4}{[x]^2 - 2[x] - 3} \Rightarrow [x] \neq -3, -1$

$Df = \mathbb{R} - \{[-1, 0) \cup [2, 3)\}$

الف) $y = \sqrt{[x - \frac{1}{x}] + [x + \frac{x}{x}]} \quad [x - \frac{1}{x}] + [x + \frac{x}{x}] \geq 0 \quad [x - \frac{1}{x} + 1] + [x + \frac{x}{x}] \geq 0$

$\sqrt{[x + \frac{x}{x}] - 1} \geq 0 \quad [x + \frac{x}{x}] \geq \frac{1}{x} \rightarrow x + \frac{x}{x} \geq 1 \quad x \geq \frac{1}{x} \rightarrow Df = [\frac{1}{x}, +\infty)$

ب) $y = \sqrt{[x - \frac{1}{x}] + [\frac{1}{x} - x]} \quad [x - \frac{1}{x}] + [-x + \frac{1}{x}] \rightarrow [a] + [-a] = 0, a \in \mathbb{Z}$
 $[a] + [-a] = -1, a \notin \mathbb{Z}$

$Df = \{x/x = k + \frac{1}{x}, k \in \mathbb{Z}\} \Rightarrow x = \frac{1}{x} \in \mathbb{Z}$

$$\text{الف) } y = \frac{\cot x + 1}{\tan x + 1}$$

$$\tan x + 1 \neq 0$$

$$\tan x \neq -1 \rightarrow x \neq \frac{3\pi}{4}, \frac{7\pi}{4}$$

$$\frac{\cos x}{\sin x} + 1$$



$$\Rightarrow \cos x \neq 0, \sin x \neq 0 \Rightarrow x \neq \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \frac{7\pi}{2}$$

$$Df = \mathbb{R} - \left\{ \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \frac{7\pi}{2} \right\}$$



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$$\text{ب) } y = \sqrt{1 - \sqrt{\sin x}} = \sqrt{(1 - \sqrt{\sin x})(1 + \sqrt{\sin x})}$$

$$|\sin x| \leq \frac{1}{4} \rightarrow |\sin x| \leq \frac{1}{4} \quad Df = \left\{ k\pi - \frac{\pi}{6}, k\pi + \frac{\pi}{6} \right\}$$



$$\text{الف) } y = \sqrt{2 \sin x - 1}$$

$$2 \sin x - 1 \geq 0$$

$$\sin x \geq \frac{1}{2}$$



$$Df = \left[\frac{\pi}{6}, \frac{5\pi}{6} \right]$$



$$\text{ب) } y = \sqrt{1 - 2 \cos x}$$

$$1 - 2 \cos x \geq 0 \rightarrow 1 \geq 2 \cos x \quad \frac{1}{2} \geq \cos x$$



$$Df = \left[\frac{\pi}{3}, \frac{2\pi}{3} \right]$$

$$\text{الف) } y = \arcsin(x^2 + 2x + 1)$$

$$-1 \leq x^2 + 2x + 1 \leq 1$$

$$x^2 + 2x + 1 \leq 1 \quad x(x+2) \leq 0 \quad \text{A}$$

$$\text{A) } \rightarrow \left[-2, 0 \right]$$

$$\text{B) } \rightarrow \left[-\infty, -2 \right] \cup \left[0, +\infty \right]$$

$$\Rightarrow Df = A \cap B = \left[-2, 0 \right]$$

$$\rightarrow \left[-2, 0 \right] \cup \left[-1, 0 \right]$$

$$x^2 + 2x + 1 \geq -1 \quad x(x+2) \geq -2 \quad \text{B}$$

$$\text{ب) } \cos y = \sqrt{5 - 2x}$$

$$\begin{cases} 5 - 2x \geq 0 & 5 \geq 2x & \frac{5}{2} \geq x & \text{A} \\ -1 \leq \sqrt{5 - 2x} \leq 1 & \rightarrow 5 - 2x \leq 1 & \rightarrow 4 \leq 2x & \rightarrow 2 \leq x & \text{B} \end{cases}$$

$$\rightarrow A \cap B = 2 \leq x \leq \frac{5}{2}$$

$$Df = \left[2, \frac{5}{2} \right]$$

$$\text{الف) } y = \log_p(1 - \log_p x)$$

$$1 - \log_p x > 0$$

$$1 > \log_p x$$

$$x > 1$$

$$Df = (0, 1)$$

$$x > 0$$

$$0 < x < p$$



$$\text{ب) } y = \log_p(1 - \log_p \frac{x}{p})$$

$$x > 0, \quad 1 - \log_p \frac{x}{p} > 0 \rightarrow 1 > \log_p \frac{x}{p} \rightarrow \frac{1}{p} > \frac{x}{p} \rightarrow x < 1$$

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$$Df = (0, \frac{1}{p}) \cup \left(\log_p \frac{1}{p}, 1 \right) \rightarrow \log_p \frac{1}{p} < 1 \rightarrow \frac{1}{p} < 1 \rightarrow Df = \left(\frac{1}{p}, 1 \right) \cup (1, +\infty)$$

$$\text{الف) } y = \sqrt{x^2 - x^2}$$

$$x^2 - x^2 \geq 0 \rightarrow 0 \geq 0$$

$$x^2 - x^2 = 0 \rightarrow (x - x)(x + x) = 0$$

$$(x - x)(x + x) = 0$$

$$+ \frac{1}{4} - \frac{3}{4}$$

$$Df = [1, 3]$$



$$\text{ب) } y = \left(\frac{x^2 + 5}{x^2 + 4} \right)!$$

$$\frac{x^2 + 5}{x^2 + 4} \in \mathbb{N}$$

$$\frac{x^2 + 5}{x^2 + 4} = \frac{x^2 + 4 + 1}{x^2 + 4} = 1 + \frac{1}{x^2 + 4}$$

$$Df = \left\{ x/x = \frac{5 - 4k}{4k - 4}, k \in \mathbb{N} \right\}$$

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$$9 - \text{ب}) \quad y = \log_y \left(1 - \log^{\frac{n}{r}} \right) \rightarrow \log^{\frac{n}{r}} \rightarrow n > .$$

$$\hookrightarrow 1 - \log^{\frac{n}{r}} > 0 \rightarrow \log^{\frac{n}{r}} < 1 \xrightarrow{\text{مناسب (مربع)}} n > \frac{1}{r}$$

$$Df_z \left(\frac{1}{r}, +\infty \right)$$