

الف) $y = \frac{x+3}{x^3+x^2-1x+3} = \frac{x+3}{(x-1)(x^2+2x-3)} = \frac{x+3}{(x-1)(x+3)(x-1)} = \frac{1}{x-1}$

$Df = \mathbb{R} - \{-3, \frac{1}{2}, 1\}$

ب) $y = \frac{x+3}{x^3+9x^2+10x+3} = \frac{x+3}{(x+1)(x^2+8x+3)} = \frac{x+3}{(x+1)(x+4)(x+1)} = \frac{1}{x+1}$

$Df = \mathbb{R} - \{-3, -1, -\frac{1}{2}\}$

الف) $y = \frac{x+3}{x^3-2x^2+x-1} = \frac{x+3}{(x-1)(x^2-x+1)} \Rightarrow Df = \mathbb{R} - \{1\}$

ب) $y = \sqrt{\frac{x+3}{x^3-2x^2+x-1}} = \sqrt{\frac{x+3}{(x-1)(x^2-x+1)}}$

$Df = (+\infty, -3] \cup (1, +\infty)$

$y = \frac{2}{x^2-5|x-1|-2x+6}$

$\rightarrow x \geq 1$

$\frac{2}{x^2-5x+6-2x+6} = \frac{2}{x^2-7x+12} = \frac{2}{(x-3)(x-4)}$

$\rightarrow x \leq 1$

$\frac{2}{x^2+5x-5-2x+6} = \frac{2}{x^2+3x+1} = \frac{2}{x(x+3)}$

$Df = \mathbb{R} - \{-3, 0, 2, 5\}$

الف) $y = \sqrt{[x]-2}$ $[x] \geq 2$ $Df = (-\infty, -2]$

ب) $y = \frac{5x^2+4}{[x]^2-2[x]-3} = \frac{5x^2+4}{([x]-3)([x]+1)} \Rightarrow [x] \neq 3, -1$

$Df = \mathbb{R} - [-1, 0) \cup [3, 4)$

الف) $y = \sqrt{[x-\frac{1}{x}]+[x+\frac{x}{x}]} \quad [x-\frac{1}{x}]+[x+\frac{x}{x}] \geq 0 \quad [x-\frac{1}{x}+1]+[x+\frac{x}{x}] \geq 0$

$\sqrt{[x+\frac{x}{x}]-1} \geq 0 \quad [x+\frac{x}{x}] \geq \frac{1}{x} \rightarrow x+\frac{x}{x} \geq 1 \quad x \geq \frac{1}{x} \rightarrow Df = [\frac{1}{x}, +\infty)$

ب) $y = \sqrt{[x-\frac{1}{x}]+[\frac{1}{x}-x]} \quad [x-\frac{1}{x}]+[-(x-\frac{1}{x})] \rightarrow [a]+[-a]=0, a \in \mathbb{Z}$

$[a]+[-a]=-1, a \notin \mathbb{Z}$

$Df = \{x/x = k+\frac{1}{x}, k \in \mathbb{Z}\} \quad \hookrightarrow x-\frac{1}{x} \in \mathbb{Z}$

$$\text{الف) } y = \frac{\cot x + 1}{\tan x + 1}$$

$$\tan x + 1 \neq 0$$

$$\tan x \neq -1 \rightarrow \omega: \pi/4, 3\pi/4 \rightarrow \frac{\pi}{4}, \frac{5\pi}{4}$$

$$\frac{\cos x}{\sin x} + 1$$



$$\Rightarrow \cos x \neq 0, \sin x \neq 0 \Rightarrow \neq \pi/2, \pi, \frac{3\pi}{2}, \frac{\pi}{2}$$

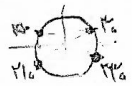
$$Df = \mathbb{R} - \left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \right\}$$



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$$\text{ب) } y = \sqrt{1 - \sqrt{2} \sin x} = \sqrt{(1 - \sqrt{2} \sin x)(1 + \sqrt{2} \sin x)}$$

$$|\sin x| \leq \frac{1}{\sqrt{2}} \rightarrow |\sin x| \leq \frac{1}{\sqrt{2}} \quad Df = \left\{ k\pi - \frac{\pi}{4}, k\pi + \frac{\pi}{4} \right\}$$



$$\text{الف) } y = \sqrt{\sqrt{2} \sin x - 1}$$

$$\sqrt{2} \sin x - 1 \geq 0$$

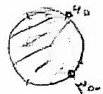
$$\sin x \geq \frac{1}{\sqrt{2}}$$



$$Df = \left[\frac{\pi}{4}, \frac{3\pi}{4} \right]$$

$$\text{ب) } y = \sqrt{1 - \sqrt{2} \cos x}$$

$$1 - \sqrt{2} \cos x \geq 0 \rightarrow 1 \geq \sqrt{2} \cos x \quad \frac{1}{\sqrt{2}} \geq \cos x$$



$$Df = \left[\frac{\pi}{4}, \frac{3\pi}{4} \right]$$

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$$\text{الف) } y = \arcsin(x^2 + 2x + 1)$$

$$-1 \leq x^2 + 2x + 1 \leq 1$$

$$\begin{aligned} x^2 + 2x + 1 &\leq 1 \\ x(x+2) &\leq 0 \end{aligned} \quad \text{A}$$

$$\text{A) } \rightarrow \begin{array}{c} -2 \\ | \\ + \end{array} \begin{array}{c} 0 \\ | \\ + \end{array}$$

$$\rightarrow [-2, 0]$$

$$\text{B) } \rightarrow \begin{array}{c} -2 \\ | \\ + \end{array} \begin{array}{c} -1 \\ | \\ + \end{array}$$

$$\rightarrow (-\infty, -2] \cup [-1, +\infty)$$

$$\Rightarrow Df = A \cap B =$$

$$[-2, -1] \cup [0, 1]$$

$$\begin{aligned} 0 &\leq x^2 + 2x + 1 \\ 0 &\leq (x+1)(x+1) \end{aligned} \quad \text{B}$$

$$\text{ب) } \cos y = \sqrt{\omega - 2x}$$

$$\begin{cases} \omega - 2x \geq 0 & \omega \geq 2x & \frac{\omega}{2} \geq x \quad \text{A} \\ -1 \leq \sqrt{\omega - 2x} \leq 1 & \omega - 2x \leq 1 & \omega \leq 2x + 1 \rightarrow \frac{\omega}{2} \leq x + \frac{1}{2} \quad \text{B} \end{cases}$$

$$\rightarrow A \cap B = \frac{\omega}{2} \leq x \leq \frac{\omega}{2} + \frac{1}{2}$$

$$Df = \left[\frac{\omega}{2}, \frac{\omega}{2} + \frac{1}{2} \right]$$

$$\text{الف) } y = \log_p(1 - \log_p x)$$

$$1 - \log_p x > 0$$

$$1 > \log_p x$$

$$x > 0$$

$$x > 0$$

$$0 < x < p$$

$$Df = (0, p)$$

$$\text{ب) } y = \log_p(1 - \log_p \frac{x}{p})$$

$$x > 0, \quad 1 - \log_p \frac{x}{p} > 0 \rightarrow 1 > \log_p \frac{x}{p} \rightarrow \frac{1}{p} > \frac{x}{p} \rightarrow x < 1$$

$$Df = (0, \frac{1}{p})$$

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$$\text{الف) } y = \sqrt{x^2 - x^2}$$

$$x^2 - x^2 \geq 0 \rightarrow x^2 \geq x^2$$

$$x^2 - x^2 = 0 \rightarrow (x^2 - x^2) = 0$$

$$(x^2 - x^2)(x^2 - x^2) = 0$$

$$\begin{array}{c} + \\ | \\ - \end{array} \begin{array}{c} 0 \\ | \\ + \end{array}$$

$$Df = [0, 1]$$

$$\text{ب) } y = \left(\frac{x^2 + \omega}{x^2 + x} \right)!$$

$$\frac{x^2 + \omega}{x^2 + x} \in \mathbb{W}$$

$$\frac{-(x^2 + \omega)}{x^2 + x} = \frac{-x^2 - \omega}{x^2 + x}$$

$$= \frac{-x^2 - \omega}{x^2 + x}$$

$$Df = \left\{ x/x = \frac{\omega - x^2}{x^2 + x}, K \in \mathbb{W} \right\}$$

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