

الف) $2x^3 + 3x^2 - 12x + 3 \mid x-1 \rightarrow 2x^2 + 5x - 3 \rightarrow (2x-1)(x+3)(x-1) \rightarrow D = \mathbb{R} - \{1, -3, \frac{1}{2}\}$

ب) $2x^3 + 9x^2 + 10x + 3 \mid x+1 \rightarrow 2x^2 + 7x + 3 \rightarrow (2x+1)(x+3)(x+1) \rightarrow D = \mathbb{R} - \{-3, -1, -\frac{1}{2}\}$

الف) $y = \frac{x+3}{x^2-2x+2x-1} = \frac{x+3}{(x-1)(x^2-2x+1)} \rightarrow D = \mathbb{R} - \{1\}$

ب) $\frac{x+3}{x^2-2x+2x-1} \rightarrow \frac{x+3}{x^2-2x+1} \rightarrow \frac{x+3}{(x-1)^2} \rightarrow D = (-\infty, -3] \cup (1, +\infty)$

$y = \frac{2}{x^2-5|x-1|-2x+5} \rightarrow x^2-5|x-1|-2x+5 \neq 0$

$x \geq 1 \rightarrow x^2-5x+5-2x+5 = x^2-7x+10 = (x-2)(x-5) \rightarrow x=2, x=5$

$x < 1 \rightarrow x^2-5(-x+1)-2x+5 = x^2+3x-5+5 = x^2+3x = x(x+3) \rightarrow x=0, x=-3$

$D_f = \mathbb{R} - \{-3, 0, 2, 5\}$

الف) $y = \sqrt{[-x]-2} \rightarrow [-x]-2 \geq 0 \rightarrow [-x] \geq 2 \rightarrow -x \geq 2 \rightarrow x \leq -2 \rightarrow D = (-\infty, -2]$

ب) $y = \frac{5x^2+4}{[x]^2-2[x]-3} \rightarrow [x]^2-2[x]-3 \neq 0 \rightarrow ([x]-3)([x]+1) \neq 0$

$[x] \neq 3 \rightarrow x \geq 3$
 $[x] \neq -1 \rightarrow x < -1$

$D = (-\infty, -1) \cup [0, 3) \cup [4, +\infty)$

الف) $y = \sqrt{[x-\frac{1}{3}]} + [x+\frac{2}{3}] \rightarrow [x-\frac{1}{3}] + [x+\frac{2}{3}] \geq 0 \rightarrow 2[x-\frac{1}{3}] + 1 \geq 0 \rightarrow [x-\frac{1}{3}] \geq -\frac{1}{2}$

$D = [\frac{1}{3}, +\infty)$

ب) $y = \sqrt{[x-\frac{1}{3}]} + [\frac{1}{3}-x] \rightarrow [x-\frac{1}{3}] + [\frac{1}{3}-x] \geq 0 \rightarrow [x] + [-x] \geq 0 \rightarrow \mathbb{Z} \rightarrow D = \{\frac{1}{3} + k, k \in \mathbb{Z}\}$

این تابع در اعداد صحیح برابر با صفر است.

$$\text{الف) } y = \frac{\cot x + 1}{\tan x + 1} \rightarrow \begin{cases} \sin x \neq 0 \\ \cos x \neq 0 \end{cases} \rightarrow \tan x + 1 \neq 0 \rightarrow \tan x \neq -1 \left\{ D = \mathbb{R} - \left\{ k\pi, \frac{\pi}{4} + k\pi, k\pi + \frac{3\pi}{4} \right\} \right. \quad k \in \mathbb{Z}$$

$$\text{ب) } y = \sqrt{1 - 4\sin^2 x} \rightarrow 1 - 4\sin^2 x \geq 0 \rightarrow \sin^2 x \leq \frac{1}{4} \rightarrow |\sin x| \leq \frac{1}{2} \rightarrow -\frac{\pi}{6} \leq x \leq \frac{\pi}{6}$$

$$D = \left[2k\pi + \frac{\pi}{6}, 2k\pi - \frac{\pi}{6} \right]$$

$$\text{الف) } y = \sqrt{2\sin x - 1} \rightarrow 2\sin x - 1 \geq 0 \rightarrow \sin x \geq \frac{1}{2}$$

$$D = \left[2k\pi + \frac{\pi}{6}, 2k\pi + \frac{5\pi}{6} \right]$$

$$\text{ب) } y = \sqrt{1 - 2\cos x} \rightarrow 1 - 2\cos x \geq 0 \rightarrow \cos x \leq \frac{1}{2}$$

$$D = \left[2k\pi + \frac{\pi}{3}, 2k\pi + \frac{5\pi}{3} \right]$$

$$\text{الف) } y = \arcsin(x^2 + 3x + 1) \rightarrow -1 \leq x^2 + 3x + 1 \leq 1 \rightarrow \begin{cases} x^2 + 3x + 1 \geq -1 \rightarrow (x+1)(x+2) \geq 0 \\ x^2 + 3x + 1 \leq 1 \rightarrow x^2 + 3x \leq 0 \rightarrow x(x+3) \leq 0 \end{cases} \quad I \cap II \Rightarrow D = [-2, -1] \cup [-1, 0]$$

$$\text{ب) } \cos y = \sqrt{5 - 2x} \rightarrow -1 \leq \sqrt{5 - 2x} \leq 1 \rightarrow 0 \leq 5 - 2x \leq 1 \rightarrow \begin{cases} 5 - 2x \geq 0 \rightarrow x \leq \frac{5}{2} \\ \sqrt{5 - 2x} \leq 1 \rightarrow 5 - 2x \leq 1 \rightarrow x \geq 2 \end{cases} \quad I \cap II \rightarrow D = \left[2, \frac{5}{2} \right]$$

$$\text{الف) } y = \log_2 \left(\frac{1 + \log_3 x}{x} \right) \rightarrow \begin{cases} x > 0 \\ 1 + \log_3 x > 0 \rightarrow \log_3 x > -1 \rightarrow x > \frac{1}{3} \\ \frac{1 + \log_3 x}{x} < 1 \rightarrow 1 + \log_3 x < x \rightarrow x > 1 \end{cases} \rightarrow D = (1, 3)$$

$$\text{ب) } y = \log_2 \left(1 - \log_2 \frac{x}{x-1} \right) \rightarrow \begin{cases} x > 0 \\ 1 - \log_2 \frac{x}{x-1} > 0 \rightarrow \log_2 \frac{x}{x-1} < 1 \rightarrow \frac{x}{x-1} < 2 \rightarrow x > \frac{1}{2} \end{cases} \quad I \cap II \rightarrow D = \left(\frac{1}{2}, +\infty \right)$$

$$\text{الف) } y = \sqrt{2^{x-x^2} - 1} \rightarrow 2^{x-x^2} - 1 \geq 0 \rightarrow 2^{x-x^2} \geq 1 \rightarrow x - x^2 \geq 0 \rightarrow -x^2 + x \geq 0 \rightarrow x^2 - x \leq 0 \rightarrow (x-1)(x-0) \leq 0 \rightarrow \frac{1}{x-1} \rightarrow D = [0, 1]$$

$$\text{ب) } y = \left(\frac{2x+5}{2x+4} \right)! \rightarrow \frac{2x+5}{2x+4} \in \mathbb{W} \rightarrow 2x+5 = W(2x+4) \rightarrow x = \frac{4W-5}{2-2W}$$

$$2x+4 \neq 0 \rightarrow x \neq -2 \quad D = \left\{ \frac{4k-5}{2-2k} \mid k \in \mathbb{W} \right\}$$