

الف) $y = \frac{x+1}{x^2+x^2-1} = \frac{x+1}{(x-1)(x^2+x+1)}$ $\rightarrow \frac{0}{0} \rightarrow \frac{1}{x} = -1$ $\rightarrow \frac{1}{x} = -1$ $\rightarrow x = -1$

$D_f = \mathbb{R} - \{-1, \frac{1}{x}, 1\}$

ب) $y = \frac{x+1}{x^2+4x^2+10x+1} = \frac{x+1}{(x+1)(x^2+4x+1)}$ $\rightarrow \frac{0}{0} \rightarrow \frac{1}{x} = -1$ $\rightarrow \frac{1}{x} = -1$ $\rightarrow x = -1$

$D_f = \mathbb{R} - \{-1, -1, -\frac{1}{x}\}$

الف) $y = \frac{x+1}{x^2-x^2+x-1} = \frac{x+1}{(x-1)(x^2-x+1)}$ $D_f = \mathbb{R} - \{1\}$

ب) $y = \sqrt{\frac{x+1}{x^2-1x^2+x-1}} = \sqrt{\frac{x+1}{(x-1)(x^2-x+1)}}$ $\frac{-1}{+1} = -1$

$D_f = (-\infty, -1] \cup (1, +\infty)$

$y = \frac{1}{x^2-2x+1-x^2+2x+1} = \frac{1}{2x^2-4x+2} = \frac{1}{2(x-1)^2}$ $\rightarrow x > 1$ $\rightarrow x < 1$

$D_f = \mathbb{R} - \{-1, 0, 1, 2\}$

الف) $y = \sqrt{[-x]-2}$ $[-x] \geq 2 \rightarrow [x] \leq -2$ $D_f = (-\infty, -2)$

ب) $y = \frac{5x^2+4}{[x]^2-2[x]-1} = \frac{5x^2+4}{([x]-1)([x]+1)}$ $\rightarrow [x] \neq 1, -1$ $D_f = \mathbb{R} - ([1, 0) \cup [1, 1))$

الف) $y = \sqrt{[x-\frac{1}{x}]+[x+\frac{1}{x}]}$ $[x-\frac{1}{x}] + [x+\frac{1}{x}] \geq 0 \rightarrow [x-\frac{1}{x}+1] + [x+\frac{1}{x}] \geq 0 \rightarrow 2[x+\frac{1}{x}]-1 \geq 0$

$[x+\frac{1}{x}] \geq \frac{1}{2} \rightarrow x+\frac{1}{x} \geq 1 \rightarrow x \geq \frac{1}{x}$ $D_f = [\frac{1}{x}, +\infty)$

ب) $y = \sqrt{[x-\frac{1}{x}]+[\frac{1}{x}-x]}$ $[x-\frac{1}{x}] + [-x+\frac{1}{x}] \rightarrow [a] + [-a] = 0 \quad a \in \mathbb{Z}$

$[a] + [a] = 1 \quad a \notin \mathbb{Z} \rightarrow x-\frac{1}{x} \in \mathbb{Z}$

$D_f = \{x/n = k + \frac{1}{n}, k \in \mathbb{Z}\}$

$\text{a)} y = \frac{\cot x + 1}{\sin x + 1}$ $\frac{\cos x}{\sin x} + 1$ $\frac{\sin x}{\cos x} + 1$ $\frac{\tan x + 1}{\tan x + 1}$ $\tan x + 1 \neq 0$ $\tan x \neq -1 \rightarrow \frac{1}{2}\pi, \frac{3}{2}\pi \rightarrow \frac{r}{2}\pi, \frac{v}{2}\pi$ $\cos x \neq 0, \sin x \neq 0 \rightarrow \neq r\pi, \pi, \frac{r}{2}\pi, \frac{v}{2}\pi$ $D_f = \mathbb{R} - \left\{ \frac{K\pi}{r}, rK\pi + \frac{v\pi}{r}, rK\pi + \frac{r\pi}{r} \right\}$ $\rightarrow y = \sqrt{1 - r \sin x} = \sqrt{(1 - r \sin x)(1 + r \sin x)}$ $D_f = \left\{ K\pi - \frac{\pi}{4}, K\pi + \frac{\pi}{4} \right\}$	6
$\text{b)} y = \sqrt{r \sin x - 1}$ $r \sin x - 1 \geq 0$ $\sin x \geq \frac{1}{r}$ $\rightarrow y = \sqrt{1 - r \cos x}$ $1 - r \cos x \geq 0$ $1 \geq r \cos x$ $\frac{1}{r} \geq \cos x$ $D_f = \left[rK\pi + \frac{\pi}{4}, rK\pi + \frac{5\pi}{4} \right]$ $D_f = \left[rK\pi + \frac{\pi}{r}, rK\pi + \frac{5\pi}{r} \right]$	7
$\text{c)} y = \arcsin(x^2 + 1)$ $-1 \leq x^2 + 1 \leq 1 \rightarrow x^2 + 1 - 1 \leq 0$ $x(x+1) \leq 0$ ① $\frac{-r}{+ - +} [-r, 0]$ ② $\frac{-r}{+ - +} (-\infty, -r] \cup [r, +\infty)$ $\Rightarrow D_f = ① \cap ② = [-r, -r] \cup [-1, 0]$ $\rightarrow y = \sqrt{a - rx}$ $a - rx \geq 0 \rightarrow a \geq rx \rightarrow \frac{a}{r} \geq x$ ③ $-1 \leq \sqrt{a - rx} \leq 1 \rightarrow a - rx \leq 1 \rightarrow r \leq rx \rightarrow r \leq x$ ④ $\Rightarrow ③ \cap ④ = r \leq x \leq \frac{a}{r} \Rightarrow D_f = [r, \frac{a}{r}]$	8
$\text{d)} y = \log_r(1 - \log_r^{\frac{1}{r}})$ $1 - \log_r^{\frac{1}{r}} > 0 \rightarrow 1 > \log_r^{\frac{1}{r}} \rightarrow r > x$ $x > 0$ $\cdot x < r$ $D_f = (0, r)$ $\rightarrow y = \log_r(1 - \log_r^{\frac{1}{r}})$ $1 - \log_r^{\frac{1}{r}} > 0 \rightarrow 1 > \log_r^{\frac{1}{r}} \rightarrow \frac{1}{r} > x$ $x > 0$ $D_f = (0, \frac{1}{r})$	9
$\text{e)} y = \sqrt{r^{x-x^2} - 1}$ $r^{x-x^2} \geq 1 = r^0$ $x - x^2 \geq 0$ $x(1-x) \geq 0$ $D_f = [0, 1]$ $\rightarrow y = \left(\frac{r^x + a}{r^x + r} \right)!$ $\frac{r^x + a}{r^x + r} \in \mathbb{N}$ $\frac{-(r^x - a)}{r^x - r} = \frac{-r^x + a}{r^x - r}$ $D_f = \left\{ x/r = \frac{-r^x + a}{r^x - r}, K \in \mathbb{N} \right\}$	10