

$$f(x) = \sqrt{(a-x)x^2 + b} \quad [1, 3, +\infty)$$

$$a=2 \quad x+b \geq 0$$

$$x=1^+ \rightarrow a+b=c \rightarrow b=-4$$

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$$f(x) = \sqrt{x^2 + 2x + 1 - m^2} \quad D_f = \mathbb{R}$$

$$x^2 + 2x - 1 - m^2 \geq 0$$

$$\Delta \leq 0 \rightarrow 4 - 4(1 - m^2) = 4 - 4 + 4m^2 = 4m^2 \leq 0 \rightarrow m^2 \leq 0 \rightarrow m = 0$$

$$m_{\max} - m_{\min} = 1 - (-1) = 2$$

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$$f(x) = \frac{\sqrt{t-x^2}}{[x] + [-x] + 1}$$

$$t-x^2 \geq 0 \rightarrow t \geq x^2 \rightarrow -\sqrt{t} \leq x \leq \sqrt{t} \quad ①$$

$$[x] + [-x] = -1 \rightarrow x \notin \mathbb{Z}$$

$$[x] + [x] \neq -1 \rightarrow x \in \mathbb{Z} \quad ②$$

$$① \cap ② \rightarrow \{-2, -1, 0, 1, 2\}$$

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$$الف) y = \sqrt{x^3 + 3x^2}$$

$$D_f = [-3, +\infty)$$

$$x^3 + 3x^2 \geq 0 \rightarrow x^2(x+3) \geq 0 \rightarrow x \geq -3$$

$$ب) y = \sqrt{\frac{(x^3-1)(x^3-13x^2)}{x^3-1}} \geq 0$$

$$x^3 \neq 1 \rightarrow x \neq 1$$

$$(x^3-1)(x^3-13x^2) \geq (x-1)(x+1)x^2(x-13)$$

$$\frac{x^2(x-1)(x+1)(x-13)}{(x-1)(x^2+x+1)} \geq 0$$

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از کجا مقید؟

$$D_f = (-\infty, -1] \cup [3, +\infty) \cup \{0\}$$

عبارت به ازای $x=0$ برابر ۰ است پس در دامنه است.

$$الف) y = \sqrt{|x-2|-3}$$

$$D_f = (-\infty, -1] \cup [5, +\infty)$$

$$|x-2| \geq 3 \rightarrow \begin{cases} x-2 \geq 3 & x \geq 5 \\ x-2 \leq -3 & x \leq -1 \end{cases}$$

$$ب) y = \sqrt{t-12-x}$$

$$D_f = [-2, 4]$$

$$t \geq |x-2| \rightarrow \begin{cases} t \geq x-2 & x \leq -2 \\ -t \leq x-2 & x \leq 4 \end{cases}$$

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الف) $\sqrt{1-x|x|}$

$1-x|x| \geq 0 \rightarrow \begin{cases} x \geq 0 \rightarrow 1-x^2 \geq 0 \rightarrow x^2 \leq 1 \rightarrow -1 \leq x \leq 1 \\ x < 0 \rightarrow 1-x(-x) \geq 0 \rightarrow 1+x^2 \geq 0 \rightarrow x < 0 \end{cases}$

$D_f = (-\infty, 1]$

ب) $\frac{1}{\sqrt{1-x|x|}}$

$1-x|x| > 0, x \neq 1$

$\rightarrow 1-x|x| \leq 1^0 \rightarrow -1 \leq x-|x| \leq 1^0 \rightarrow -1 \leq x \leq 1$

$1-x|x| \neq 1 \rightarrow x \neq 1^0$

$D_f = [1, 1] - \{1\} = \emptyset$

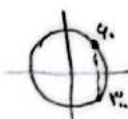
الف) $y = \frac{\sin^n x - 1}{\cos^n x}$

$\cos x \neq 0 \rightarrow D_f = \mathbb{R} - \{2K\pi + \frac{\pi}{2}, 2K\pi + \frac{3\pi}{2}\}$



ب) $\frac{\cos^n x}{\sqrt{\cos^n x - 1}}$

$\sqrt{\cos^n x - 1} \neq 0 \rightarrow \cos^n x \neq 1 \rightarrow \cos x \neq \pm 1$



$D_f = \mathbb{R} - \{2K\pi + \frac{\pi}{2}, 2K\pi + \frac{3\pi}{2}\}$

الف) $y = \frac{x+1^0}{11x+1-|x+1^0|}$

$|x+1|-|x+1^0| \neq 0$
 $|x+1| \neq |x+1^0|$

$\begin{cases} x+1 = x+1^0 \rightarrow x-x = 1-1 \rightarrow x=0 \\ x+1 = -(x+1^0) \rightarrow x+1 = -x-1 \rightarrow x+x = -1-1 \rightarrow x = -1 \end{cases}$

$D_f = \mathbb{R} - \{-\frac{1}{11}, 1\}$

ب) $y = \sqrt{|x+1|-|x+1^0|}$

$x \geq -1 \wedge x \leq -\frac{1}{11}$

$|x+1|-|x+1^0| \geq 0$
 $|x+1| \geq |x+1^0|$

$\begin{cases} (x+1)^2 \geq (x+1^0)^2 \rightarrow x^2 + 2x + 1 \geq x^2 + 2x + 1 \\ \rightarrow x^2 - x - 1 \geq 0 \\ \Delta = (-1)^2 - 4(-1) = 5 \\ x = \frac{1 \pm \sqrt{5}}{2} \end{cases}$

$D_f = (-\infty, -\frac{1}{11}] \cup [\frac{1+\sqrt{5}}{2}, +\infty)$

$f(x) = \sqrt{\log_{10} \log_{10} (x-1)}$

$x-1 > 0 \rightarrow x > 1 \rightarrow x > \frac{1}{10}$

$\log_{10} (x-1) > 1 \rightarrow x-1 > 10 \rightarrow x > 11$

$\rightarrow x > 11 \rightarrow x > 10 \rightarrow x > 10$

وقتی متغیرها و اعداد حقیقی نامعادله عوض می‌شود
(معمولاً نتیجه می‌دهد!)

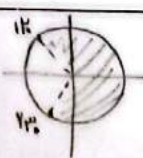
$\textcircled{1} \cap \textcircled{2} = (11, +\infty)$

الف) $\log (\cos^n x + 1)$

$\cos^n x + 1 > 0$

$\cos^n x > -1$

$\cos x > -\frac{1}{10}$



$D_f = (2K\pi - \frac{\pi}{10}, 2K\pi + \frac{\pi}{10})$

ب) $y = \sqrt{\log \frac{x-1}{x+1}}$

$\frac{x-1}{x+1} > 0, \log \frac{x-1}{x+1} > 0 \rightarrow \frac{x-1}{x+1} > 1 \rightarrow \frac{x-1}{x+1} - 1 > 0$

$\frac{x-1-x-1}{x+1} > 0 \rightarrow \frac{-2}{x+1} > 0 \rightarrow x+1 < 0 \rightarrow x < -1$

$D_f = (-\infty, -1)$

$$4- \text{ب}) \frac{(n^2-1)(n^2-n^2)}{(n^2-1)} \geq 0 \rightarrow \frac{(n-1)(n+1)(n^2)(n-1)}{(n-1)(n^2+n+1)} \geq 0$$

$$D_f = (-\infty, -1] \cup \{0\} \cup [3, +\infty)$$

$$\begin{array}{ccccccc} & -1 & & 0 & & 1 & & 3 \\ & + & & - & & - & & + \end{array}$$

$$9) \log_{\Delta} \log_{\Delta}^{n-1} \rightarrow n-1 > 0 \rightarrow n > \frac{1}{\Delta}$$

$$\log_{\Delta}^{n-1} > 0 \rightarrow n-1 > \Delta^0 \rightarrow n-1 > 1 \rightarrow n > 2$$

$$\log_{\Delta} \log_{\Delta}^{n-1} \geq 0 \xrightarrow[\text{من باب عوض!}]{\text{من بابی در!}} \log_{\Delta}^{n-1} \leq \frac{1}{\Delta} \rightarrow \log_{\Delta}^{n-1} \leq 1 \rightarrow n-1 \leq \Delta \rightarrow n \leq 4 \rightarrow n \leq 3$$

$$\hookrightarrow D_f = (1, 3]$$