

$$f(x) = \sqrt{(a-2)x^2 + ax + b} \rightarrow \text{معادله}$$

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$$\begin{aligned} a-2 &\geq 0 \\ a &\geq 2 \\ ax+b &\rightarrow x \geq 3 \rightarrow 3a+b \geq 0 \rightarrow 3a \geq -b \\ 3(2) &\geq -b \\ b &\geq -6 \end{aligned}$$

$$f(x) = \sqrt{x^2 + 2x + 2 - m^2} \quad D_f = \mathbb{R}$$

$$\begin{aligned} \Delta &\leq 0 \rightarrow f - f(1)(2-m^2) < 0 \\ f - 1 + fm^2 &\leq 0 \rightarrow fm^2 - f < 0 \\ f(m^2-1) &\leq 0 \rightarrow [1, 1] \\ \pm 1 &\rightarrow \frac{-1 \pm 1}{2} \\ m_{\max} - m_{\min} &= 1 - (-1) = 2 \end{aligned}$$

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$$f(x) = \frac{\sqrt{4-x^2}}{[x] + [-x] + 1}$$

$$\begin{aligned} f - x^2 \geq 0 &\rightarrow f \geq x^2 \rightarrow [-2, 2] \\ [x] + [-x] + 1 &\neq 0 \rightarrow [x] + [-x] \neq -1 \rightarrow \mathbb{Z} \end{aligned} \quad \left. \begin{aligned} & \\ & \end{aligned} \right\} \cap \rightarrow D_f = [-2, 2] \rightarrow -2, -1, 0, 1, 2 \rightarrow \infty$$

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$$\text{الف) } y = \sqrt{x^3 + 3x^2}$$

$$x^3 + 3x^2 \geq 0 \rightarrow x^2(x+3) \geq 0$$

$$\frac{x^3}{-} \quad \frac{-3}{+} \quad D_f = [-3, +\infty)$$

$$\text{ب) } y = \sqrt{\frac{(x^2-1)(x^3-3x^2)}{x^3-1}}$$

$$\frac{(x^2-1)(x^3-3x^2)}{(x-1)(x^2+x+1)} \geq 0 \rightarrow \frac{(x-1)(x+1)(x^2-3x)}{(x-1)(x^2+x+1)} \geq 0$$

$$\frac{-1}{+} \quad \frac{0}{-} \quad \frac{3}{-} \quad \frac{+}{+} \quad D_f = (-\infty, -1] \cup [3, +\infty) \cup \{0\}$$

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$$\text{الف) } y = \sqrt{|x-2|-3}$$

$$|x-2|-3 \geq 0 \rightarrow |x-2| \geq 3 \rightarrow \begin{aligned} x-2 &\geq 3 \rightarrow x \geq 5 \\ -x+2 &\geq 3 \rightarrow x \leq -1 \end{aligned}$$

$$D_f = (-\infty, -1] \cup [5, +\infty)$$

$$\text{ب) } y = \sqrt{4-|2-x|}$$

$$\begin{aligned} 4-|2-x| &\geq 0 \rightarrow 4 \geq |2-x| \rightarrow -4 \leq 2-x \leq 4 \\ -4 &\leq -x \leq 2 \\ 4 &\geq x \geq -2 \end{aligned}$$

$$D_f = [-2, 4]$$

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