

الف)  $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} = \sqrt{\frac{(x-1)^2 - x^2}{x(x-1)}} = \sqrt{\frac{1-2x}{x(x-1)}} = \frac{1-2x}{x(x-1)} > 0$

$\frac{0}{+} \frac{1}{-} \frac{1}{+} \frac{1}{-}$   $D_f = (-\infty, 0) \cup \left[\frac{1}{2}, 1\right)$

ب)  $f(x) = \frac{\frac{1}{x+1} - \frac{x}{x-1}}{\frac{3}{x-1} + \frac{1}{x+1}} \rightarrow x+1 \neq 0 \Rightarrow x \neq -1 \mid x-1 \neq 0 \Rightarrow x \neq 1 \mid x+1 \neq 0 \Rightarrow x \neq -1 \mid x-1 \neq 0 \Rightarrow x \neq 1$

$\frac{3}{x-1} + \frac{1}{x+1} \neq 0 \Rightarrow \frac{3x+1+x-1}{(x-1)(x+1)} \neq 0 \Rightarrow x \neq -\frac{1}{4}, 1, -2 \Rightarrow D_f = \mathbb{R} - \left\{1, -2, -\frac{1}{4}\right\}$

الف)  $f(x) = \sqrt{\left(\frac{1}{3}\right)^x - 9} (3^x - 4^x) \Rightarrow \left[\left(\frac{1}{3}\right)^x - 9\right] (3^x - 4^x) > 0 \Rightarrow \frac{-2}{+} \frac{0}{+} \frac{0}{+}$

$D_f = [-2, \frac{0}{3}]$

منها اگر بماند: به هم جواب ندهد + است!  $\left(\frac{1}{3}\right)^x - 9 > 0 \Rightarrow \left(\frac{1}{3}\right)^x > 9 \Rightarrow \left(\frac{1}{3}\right)^x > 3^2 \Rightarrow \left(\frac{1}{3}\right)^x > \left(\frac{1}{3}\right)^{-2} \Rightarrow x < -2$

ب)  $\sqrt{x-1} + \sqrt{y+1} = 3 \Rightarrow x-1 \geq 0 \Rightarrow x \geq 1 \Rightarrow D_f = [1, +\infty)$   $\sqrt{y+1} = 3 - \sqrt{x-1} \Rightarrow y+1 = 9 - 6\sqrt{x-1} + x-1 \Rightarrow y = x - 6\sqrt{x-1} + 7$

$f(x) = \log_p(x^2 - x - 2) \rightarrow x^2 - x - 2 > 0 \Rightarrow (x-2)(x+1) > 0 \Rightarrow \frac{-1}{+} \frac{2}{+}$

$\sqrt{x^2 - 1} + 1 \rightarrow x^2 - 1 > 0 \Rightarrow (x-1)(x+1) > 0 \Rightarrow \frac{-1}{+} \frac{1}{+} \mid \sqrt{x^2 - 1} + 1 \neq 0 \Rightarrow$  معادله صحیح

$\Rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$

$f(x) = \sqrt{2 + ax - x^2} \rightarrow x = 1 \mid f(x) = 0 \Rightarrow \sqrt{2 - 1a + 1} = 0 \Rightarrow 1 - 1a = 0 \Rightarrow a = \frac{1}{1}$

$D_f = [-2, b]$   $a = \frac{1}{1}, b = \frac{3}{1} \Rightarrow a+b=1$   $f(x) = \sqrt{2 - \frac{1}{1}x - x^2} = \sqrt{-(x+1)(x-\frac{3}{1})}$

$f(x) = \begin{cases} 3x-1 & x \geq 1 \\ 3x+3 & x \leq 1 \end{cases} \quad g(x) = \sqrt{f(x)-x} \Rightarrow f(x)-x \geq 0$

$\Rightarrow D_g = [2, +\infty)$   $x \geq 1 \Rightarrow 3x-1-x \geq 0 \Rightarrow x \geq \frac{1}{2} \checkmark$   
 $x \leq 1 \Rightarrow 3x+3-x \geq 0 \Rightarrow x \geq -3$   
 $\Rightarrow x \geq -3$

$f(x) = 2x^2 - 3x$

$f(\sin \frac{\pi}{4} + \cos \frac{\pi}{4}) = ?$   
 $\sin \frac{\pi}{4} + \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2} + 1 = \frac{\sqrt{2} + 2}{2}$   
 $2 \times \left(\frac{\sqrt{2} + 2}{2}\right)^2 - 3 \left(\frac{\sqrt{2} + 2}{2}\right) = \frac{2(\sqrt{2} + 2)^2}{2} - \frac{3(\sqrt{2} + 2)}{2} = \frac{2(2 + 4\sqrt{2} + 4) - 3\sqrt{2} - 6}{2} = \frac{2(6 + 4\sqrt{2}) - 3\sqrt{2} - 6}{2} = \frac{12 + 8\sqrt{2} - 3\sqrt{2} - 6}{2} = \frac{6 + 5\sqrt{2}}{2}$

$$f(x) = \frac{x+1}{x-1} \rightarrow f(a) = \frac{a+1}{a-1} \Rightarrow a+1 \rightarrow f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{\frac{a+1+a-1}{a-1}}{\frac{a+1-a+1}{a-1}} = \frac{2a}{2} = a \rightarrow \frac{a+1}{a-1} - 1 \neq a$$

$$f(f(a)) = a \Rightarrow a \in \mathbb{R} - \{1\} \quad \text{D}$$

$$\left. \begin{aligned} f(x) &= x^2 - 1 \\ g(x) &= \sqrt{x^2 - 1} \end{aligned} \right\} \Rightarrow g(a) = \sqrt{a^2 - 1}, f(\sqrt{a^2 - 1}) = a^2 - 1 - \sqrt{a^2 - 1} = -1 \Rightarrow a^2 - 1 - \sqrt{a^2 - 1} + 1 = 0$$

$$f(g(a)) = -1 \Rightarrow \sqrt{a^2 - 1} = t \Rightarrow t^2 - \sqrt{t^2} + 1 = 0 \Rightarrow t^2 - t + 1 = 0 \Rightarrow t = 1 \Rightarrow \sqrt{a^2 - 1} = 1 \Rightarrow a^2 - 1 = 1 \Rightarrow a^2 = 2 \Rightarrow a = \pm \sqrt{2}$$

$$\left. \begin{aligned} f(x+y) &= x^2 - \omega x + y \\ f(-y) &= x^2 - \omega(-y) = -1 \Rightarrow x = -y \\ f(1) &= x^2 - \omega x + 1 = 1 \Rightarrow x = 1 \end{aligned} \right\} \rightarrow f(f) \times f(-1) = (1 - \omega + 1)(1 + \omega + 1) = 1 + \omega^2$$

$$f(\cos^2 x + \omega \cos x) = \tan^2 x$$

$$f(1, \omega) = ?$$

$$\Rightarrow \cos^2 x + \omega \cos x = 1, \omega \Rightarrow x = \frac{\pi}{2} \Rightarrow \tan^2 \frac{\pi}{2} = \omega \quad \text{D}$$

$$\text{الف - 2)} \left( \left( \frac{1}{p} \right)^n - 9 \right) (32 - \varepsilon^n) \geq 0 \rightarrow \frac{-2}{+0} \frac{0}{-0} \rightarrow D_f = (-\infty, -2] \cup \left[ \frac{8}{9}, +\infty \right)$$

$$\left( \frac{1}{p} \right)^n = 9 \rightarrow n = -2 \quad 32 = \varepsilon^n \rightarrow n = \frac{5}{2}$$

$$\text{ب - 2)} \sqrt{x-1} + \sqrt{y+1} = 3, \quad \sqrt{y+1} = 3 - \sqrt{x-1} \rightarrow 3 - \sqrt{x-1} \geq 0 \rightarrow \sqrt{x-1} \leq 3 \rightarrow x \leq 10$$

$$x-1 \geq 0 \rightarrow x \geq 1 \quad \text{I}$$

$$\text{I, II} \rightarrow 1 \leq x \leq 10 \rightarrow D_f = [1, 10]$$