

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} = \sqrt{\frac{(x-1)^2 - x^2}{x(x-1)}} = \sqrt{\frac{1-2x}{x(x-1)}} = \frac{1-2x}{x(x-1)} > 0$

$\frac{0}{+} \frac{1}{-} \frac{1}{+}$ $D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $f(x) = \frac{\frac{1}{x+1} - \frac{x}{x-1}}{\frac{x}{x-1} + \frac{1}{x+1}} \rightarrow x+1 \neq 0 \Rightarrow x \neq -1$
 $x-1 \neq 0 \Rightarrow x \neq 1$
 $x+1 \neq 0 \Rightarrow x \neq -1$
 $x-1 \neq 0 \Rightarrow x \neq 1$

$\frac{x}{x-1} + \frac{1}{x+1} \neq 0 \Rightarrow \frac{x(x+1) + x-1}{(x-1)(x+1)} \neq 0 \Rightarrow x \neq -\frac{2}{3}, 1, -1 \Rightarrow D_f = \mathbb{R} - \{-1, 1, -\frac{2}{3}\}$

الف) $f(x) = \sqrt{(\frac{1}{x})^2 - 9} = \sqrt{\frac{1}{x^2} - 9} \Rightarrow (\frac{1}{x^2} - 9) \geq 0 \Rightarrow \frac{-2}{-9} + \frac{0}{-}$

$D_f = [-\frac{2}{9}, \frac{0}{9}]$

ب) $\sqrt{x-1} + \sqrt{y+1} = 3 \Rightarrow x-1 \geq 0 \Rightarrow x \geq 1 \Rightarrow D_f = [1, +\infty)$

$f(x) = \log_p(x^2 - x - 2) \rightarrow x^2 - x - 2 \geq 0 \Rightarrow (x-2)(x+1) \geq 0 \Rightarrow \frac{-1}{+} \frac{2}{+}$

$\sqrt{x^2 - 1} + 1 \rightarrow x^2 - 1 \geq 0 \Rightarrow (x-1)(x+1) \geq 0 \Rightarrow \frac{-1}{+} \frac{1}{+} / \sqrt{x^2 - 1} + 1 \neq 0 \Rightarrow$ معادله صحیح

$\Rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$

$f(x) = \sqrt{2 + ax - x^2} \rightarrow x = 1 \Rightarrow f(x) = \sqrt{2 - 1 + a} = 0 \Rightarrow -1 + a = 0 \Rightarrow a = 1$

$D_f = [-2, b]$ $a = \frac{1}{2}, b = \frac{3}{2} \Rightarrow a+b=1$

$f(x) = \sqrt{2 - \frac{1}{2}x - x^2} = \sqrt{-(x+\frac{1}{2})(x-\frac{3}{2})}$

$f(x) = \begin{cases} 2x-1 & x \geq 1 \\ 2x+3 & x \leq 1 \end{cases}$ $g(x) = \sqrt{f(x)-x} \Rightarrow f(x)-x \geq 0$
 $x \geq 1 \Rightarrow 2x-1-x \geq 0 \Rightarrow x \geq 1$
 $x \leq 1 \Rightarrow 2x+3-x \geq 0 \Rightarrow x \geq -3$
 $\Rightarrow D_g = [-3, +\infty)$

$f(x) = 2x^2 - 3x$

$f(\sin \frac{\pi}{4} + \cos \frac{\pi}{4}) = ?$

$\sin \frac{\pi}{4} + \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2} + 1 = \frac{1+\sqrt{2}}{2}$

$2 \times (\frac{1+\sqrt{2}}{2})^2 - 3 \times (\frac{1+\sqrt{2}}{2}) = \frac{2(1+2\sqrt{2}+2)}{2} - \frac{3(1+\sqrt{2})}{2} = \frac{1}{2}$

$$f(x) = \frac{x+1}{x-1} \rightarrow f(a) = \frac{a+1}{a-1} \Rightarrow a+1 \rightarrow f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{a+1}{a-1} + 1 = \frac{a+1}{a-1} + \frac{a-1}{a-1} = \frac{a+1+a-1}{a-1} = \frac{2a}{a-1}$$

$$f(f(a)) = a \Rightarrow \frac{2a}{a-1} = a \Rightarrow 2a = a(a-1) \Rightarrow 2a = a^2 - a \Rightarrow a^2 - 3a = 0 \Rightarrow a(a-3) = 0 \Rightarrow a = 0 \text{ or } a = 3$$

$$\Rightarrow a \in \mathbb{R} - \{1\}$$

$$\left. \begin{array}{l} f(x) = x^2 - 1 \\ g(x) = \sqrt{x^2 - 1} \\ f(g(a)) = -1 \end{array} \right\} \Rightarrow g(a) = \sqrt{a^2 - 1}, f(\sqrt{a^2 - 1}) = a^2 - 1 - 1 = -1 \Rightarrow a^2 - 1 - 1 = -1 \Rightarrow a^2 - 1 = 1 \Rightarrow a^2 = 2 \Rightarrow a = \pm \sqrt{2}$$

$$\left. \begin{aligned} f(x+1) &= x^2 - \omega x + 9 \\ f(-1) &\rightarrow x+1 = -1 \Rightarrow x = -2 \\ f(1) &\rightarrow x+1 = 1 \Rightarrow x = 0 \end{aligned} \right\} \rightarrow f(f) \times f(-1) = (1 - \omega + 9)(4 + 1 + 9) = \underline{144}$$

$$\rightarrow \psi \cos' x + \omega \cos x = \psi, \omega \Rightarrow x = \frac{\pi}{\mu} \Rightarrow \tan' \frac{\pi}{\mu} = \underline{\underline{\mu}}$$