

الف) $f(x) = \sqrt{\frac{x-1}{x}} - \frac{x}{x-1}$

ب) $f(x) = \frac{\frac{1}{x+1} - \frac{x}{x-1}}{\frac{x}{x-1} + \frac{1}{x+2}}$

$\frac{x-1}{x} - \frac{x}{x-1} \geq 0 \Rightarrow \frac{(x-1)^2 - x^2}{x(x-1)}$

شرط صحت: $x+1 \neq 0, x \neq 0$

شرط خارج: $\frac{x}{x-1} + \frac{1}{x+2} \neq 0 \Rightarrow \frac{x(x+2) + x-1}{(x-1)(x+2)} = \frac{x^2+3x-1}{(x-1)(x+2)}$

$x-1 \neq 0, x+2 \neq 0, x^2+3x-1 \neq 0$
 $x \neq 1, x \neq -2, x \neq \frac{-3 \pm \sqrt{17}}{2}$

$OP = (-\infty, 0) \cup (\frac{1}{2}, +\infty)$

$OP = \mathbb{R} - \{ -\frac{3}{2}, -\frac{5}{2}, -1, 0, 1 \}$

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الف) $f(x) = \sqrt{(\frac{1}{x})^2 - 9} (x^2 - 4^x)$

ب) $\sqrt{x-1} + \sqrt{y+1} = 3$

$(\frac{1}{x})^2 - 9 \geq 0 \Rightarrow x^2 - 9 \leq 0$

شرط زیر رادیکال: $x-1 \geq 0, y+1 \geq 0$

$x^2 - 9 = 0 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$

$\sqrt{y+1} = 3 - \sqrt{x-1}$

محدود: $\sqrt{y+1} \geq 0, \sqrt{x-1} \geq 0$

$x^2 - 4^x = 0 \Rightarrow x^2 = 4^x \Rightarrow x = \frac{2}{x}$

$x - \sqrt{x-1} \geq 0$

$x \geq \sqrt{x-1}$

$\textcircled{1} \cap \textcircled{2} = [1, 8^2]$

$OP = [1, 8^2]$

$8^2, x-1 \rightarrow 8^2, x$

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$f(x) = \frac{\log_2(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$

از آنجا که $\sqrt{x^2 - 1} + 1 \neq 0 \Rightarrow \sqrt{x^2 - 1} \geq 0$ است
بنابراین $\sqrt{x^2 - 1} + 1 \geq 1$

شرط صحت: $x^2 - x - 2 > 0 \Rightarrow x < -1$ یا $x > 2$

شرط زیر رادیکال: $x^2 - 1 \geq 0 \Rightarrow x \leq -1$ یا $x \geq 1$

$\frac{-1}{x} + \frac{1}{x} \rightarrow \textcircled{3} = (-\infty, -1) \cup (1, +\infty)$

$\textcircled{1} \cap \textcircled{2} = (-\infty, -1) \cup (2, +\infty)$

$OP = (-\infty, -1) \cup (2, +\infty)$

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$f(x) = \sqrt{-x^2 + ax + 3}$



$f(x) = \sqrt{-x^2 - \frac{1}{x}x + 3}$

$OP = [-2, b]$

$OP = [-2, \frac{3}{2}]$

$-x^2 + ax + 3 \geq 0$

$a + b = -\frac{1}{x} + \frac{3}{x} = 1$

ب) $a(x+2)(x-b) = -x^2 + ax + 3 \Rightarrow a(x+2)(x-b) = -x^2 + ax + 3$

$-x^2 + (b+2)x + 2b = -x^2 + ax + 3$

$a = \frac{3}{x} - \frac{1}{x} = \frac{2}{x}$

$2b = 3 \Rightarrow b = \frac{3}{2}$

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$f(x) = \begin{cases} 3x-2 & x \geq 1 \\ 2x+3 & x < 1 \end{cases}$

$OP = [1, +\infty) \cup [-3, 1) = [-3, +\infty)$

$g(x) = \sqrt{f(x) - x}$

تمام این بازه قابل قبول است

شرط زیر رادیکال: $f(x) - x \geq 0$

$\begin{cases} \text{اگر } x \geq 1 \rightarrow 3x-2-x \geq 0 \rightarrow 2x \geq 2 \Rightarrow x \geq 1 \\ \text{اگر } x < 1 \rightarrow 2x+3-x \geq 0 \Rightarrow x \geq -3 \end{cases}$

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$$f(x) = x^2 - 4x$$

$$f\left(\sin \frac{\pi}{4} + \cos \frac{\pi}{4}\right) = ?$$

$$\sin \frac{\pi}{4} + \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2} + 1 = \frac{2+\sqrt{2}}{2}$$

$$f\left(\frac{2+\sqrt{2}}{2}\right) = \left(\frac{2+\sqrt{2}}{2}\right)^2 - 4\left(\frac{2+\sqrt{2}}{2}\right)$$

$$f\left(\frac{2+\sqrt{2}}{2}\right) = \frac{2(2+\sqrt{2}+2)}{2} - 2(2+\sqrt{2})$$

$$f\left(\frac{2+\sqrt{2}}{2}\right) = \frac{4+4\sqrt{2}}{2} - 2(2+\sqrt{2})$$

$$f\left(\frac{2+\sqrt{2}}{2}\right) = \frac{4+4\sqrt{2}-4-4\sqrt{2}}{2} = -\frac{1}{2}$$

$$f\left(\sin \frac{\pi}{4} + \cos \frac{\pi}{4}\right) = -\frac{1}{2}$$

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$$f(x) = \frac{x+1}{x-1}, \quad f(f(a)) = a$$

$$f(a) = \frac{a+1}{a-1} \rightarrow a \neq 1 \text{ مخرجی ندارد}$$

$$f(f(a)) = f\left(\frac{a+1}{a-1}\right) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{a+1+a-1}{a+1-a} = \frac{2a}{1} = a$$

$$D_f = \mathbb{R} - \{1\}$$

این تابع معادله همگونی
دارد تابع برقرار است و مشتق
این است که مشتق تابع اول
در دنباله $a \neq 1$

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$$f(x) = x^2 - 4x$$

$$g(x) = \sqrt{x^2 - 1}$$

$$f(g(a)) = -4$$

$$f(g(a)) = (g(a))^2 - 4g(a) = -4$$

$$(g(a))^2 - 4g(a) + 4 = 0$$

$$(g(a) - 2)^2 = 0 \Rightarrow g(a) = 2$$

$$g(a) = 2$$

$$2 = \sqrt{a^2 - 1} \rightarrow$$

چون طرفین معادله را
مربع می‌کنیم داریم
 $4 = a^2 - 1$
و داریم $a^2 = 5$
پس $a = \pm \sqrt{5}$

$$f = a^2 - 1$$

$$a^2 = 5 \Rightarrow a = \pm \sqrt{5}$$

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$$f(x^2) = x^2 - ax + 4$$

$$f(4) \times f(-4)$$

$$x^2 = 4 \rightarrow x = 2$$

$$f(4) = (1)^2 - a(1) + 4 = 2$$

$$x^2 = -4 \rightarrow x = -2$$

$$f(-4) = (-1)^2 - a(-1) + 4 = 5$$

$$f(4) \times f(-4) = 2 \times 5 = 10$$

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$$f(\cos^2 x + a \cos x) = \tan^2 x$$

$$f(r, a) = ?$$

$$\cos^2 x + a \cos x = r, a$$

$$\cos^2 x + a \cos x - r, a = 0$$

در $\cos^2 x$

$$\cos^2 x + a \cos x - 1 = 0$$

$$(\cos x + 1)(\cos x - 1) = 0$$

$$\cos x = \frac{1}{2} \Rightarrow x = 60^\circ$$

$$f(1, a) = \tan^2 60^\circ$$

$$f(r, a) = (r, a)$$

$$f(1, a) = 3$$

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$$\left\{ \begin{array}{l} \cos x = -\frac{1}{2} \Rightarrow x = 120^\circ \\ \cos x = \frac{1}{2} \Rightarrow x = 60^\circ \end{array} \right.$$