

$$D_f = (-\infty, 0) \cup \left(\frac{1}{2}, 1\right)$$

الف) $f(x) = \sqrt{\frac{x-1}{x}} - \frac{x}{x-1}$

$$\frac{(x-1)^2 - x^2}{(x)(x-1)} = \frac{x^2 + 1 - 2x - x^2}{x^2 - x} = \frac{1 - 2x}{(x)(x-1)}$$

$$\frac{0}{+\frac{1}{2} - \frac{1}{2} + \frac{1}{2} -}$$

ب) $f(x) = \frac{1}{x+1} - \frac{2}{x}$

$$(1) : x-1 \neq 0 \Rightarrow x \neq 1$$

$$(2) : x+2 \neq 0 \Rightarrow x \neq -2$$

(3) $\frac{3}{x-1} + \frac{1}{x+2} = 0 \Rightarrow \frac{3}{x-1} = -\frac{1}{x+2} \Rightarrow 3x+4 = -x+1 \Rightarrow 4x = -3 \Rightarrow x = -\frac{3}{4}$

(4) $x+1 \neq 0 \Rightarrow x \neq -1$

$$D_f = \mathbb{R} - \{-2, -\frac{3}{4}, 0, 1\}$$

(5) $x \neq 0$

الف) $f(x) = \sqrt{\left(\frac{1}{x}\right)^2 - 9} (32 - x^2)$

$$D_f = (-\infty, -\frac{1}{3}] \cup [\frac{1}{3}, +\infty)$$

$$x=2 \text{ ریشه } \leftarrow \text{ ریشه } \Rightarrow x \geq \frac{1}{3}$$

ب) $\sqrt{x-1} + \sqrt{y+1} = 3 \Rightarrow \sqrt{y+1} = 3 - \sqrt{x-1} \Rightarrow 3 \geq \sqrt{x-1} \Rightarrow 9 \geq x-1 \Rightarrow x \leq 10$

(II) $x-1 \geq 0 \Rightarrow x \geq 1$

$$D_f = [1, 10]$$

$f(x) = \log_{\frac{x-x-2}{\sqrt{x-1}+1}}$

مخرج عبارت همواره مثبت است و منفی شود

(1) $x^2 - x - 2 > 0 \Rightarrow (x-2)(x+1) > 0 \Rightarrow \frac{-1}{+\frac{1}{2} - \frac{1}{2} +} \Rightarrow x < -1, x > 2$

(2) $x^2 - 1 > 0 \Rightarrow x^2 \geq 1 \Rightarrow x \geq 1, x \leq -1$

(1) $\cap$ (2) $\Rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$



$$f(x) = \sqrt{x^2 + ax - x^2}$$

$$[-2, b]$$

با توجه به دامنه عبارت $\sqrt{\quad}$ می توان گفت (۲) ریشه عبارت
غیر رادیکال می باشد.

$$x = -2 \rightarrow x^2 - 2ax + x^2 = 0 \rightarrow 2a = -1 \Rightarrow a = -\frac{1}{2}$$

$$-x^2 - \frac{1}{2}x + x^2 \xrightarrow{\text{مجموع دوتا}} \frac{1}{2} \pm \sqrt{\frac{1}{4} + 1} \Rightarrow \frac{1}{2} \pm \frac{\sqrt{5}}{2} \rightarrow (-2)$$

$$\frac{\frac{1}{2} + \frac{\sqrt{5}}{2}}{-2} \rightarrow \left(\frac{\frac{1}{2} + \frac{\sqrt{5}}{2}}{-2}\right) = b$$

$$a + b \Rightarrow \left[-\frac{1}{2} + \frac{\sqrt{5}}{2}\right] = 1$$

$$f(x) = \begin{cases} x^2 - 1 & x \geq 1 \\ x^2 + 3 & x < 1 \end{cases}$$

$$g(x) = \sqrt{f(x) - x} \Rightarrow f(x) \geq x \xrightarrow{(1)} x^2 - 1 \geq x \Rightarrow x^2 - x - 1 \geq 0 \Rightarrow x \geq 1$$

$$\xrightarrow{\text{افتراق با ۱}} x \geq 1$$

$$\xrightarrow{(2)} x^2 + 3 \geq x \Rightarrow x \geq -3 \xrightarrow{\text{افتراق با ۱}} -3 \leq x < 1$$

$$(1) \cup (2) \Rightarrow \bigcup_f \Rightarrow [-3, +\infty)$$

$$f(x) = x^2 - x \Rightarrow \frac{x(\sqrt{x^2 + 1})}{x} - \frac{x(\sqrt{x^2 + 1})}{x} = \frac{x + x\sqrt{x^2 + 1}}{x} = \frac{x\sqrt{x^2 + 1}}{x} = \sqrt{x^2 + 1} - 1$$

$$\Rightarrow \left[-\frac{1}{2}\right]$$

$$f\left(\sin \frac{\pi}{3} + \cos \frac{\pi}{3}\right)$$

$$\Rightarrow \sin \frac{\pi}{3} + \cos \frac{\pi}{3} = \frac{\sqrt{3}}{2} + 1 \Rightarrow \frac{\sqrt{3} + 2}{2}$$

$$f(n) = \frac{n+1}{n-1}$$

$$f(f(a)) = a \Rightarrow f(a) = \frac{a+1}{a-1}$$

مرحله اول: (V)

$$f(f(a)) \Rightarrow \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{\frac{a+1+a-1}{a-1}}{\frac{a+1-a+1}{a-1}} = \frac{2a}{2} = a$$

پسندید:

یعنی $f(f(a))$ برای هر $a \in \mathbb{R}$ در دامنه تابع با a برابر است.

$$a \neq 1 \Rightarrow f(f(a)) \neq 1 \Rightarrow a \neq 1$$

$$a \in \mathbb{R}, a \neq 1$$

$$f(x) = x^2 - 5x$$

$$g(x) = \sqrt{x^2 - 1}$$

$$f(g(a)) = -5 \Rightarrow a^2 - 5a = -5 \Rightarrow a^2 - 5a + 5 = 0 \Rightarrow (a-2)^2 = 0 \Rightarrow a = 2$$

$$g(a) = 2 \Rightarrow \sqrt{a^2 - 1} = 2 \Rightarrow a^2 - 1 = 4 \Rightarrow a^2 = 5 \Rightarrow a = \pm \sqrt{5}$$

$$f(x) \times f(-x)$$

$$f(x+1) = x^2 - 5x + 4$$

$$x+1 = 5 \Rightarrow x = 4 \Rightarrow 1 - 5 + 4 = 0$$

$$x+1 = -5 \Rightarrow x = -6 \Rightarrow 36 - 30 + 4 = 10$$

$$\left. \begin{array}{l} 10 \times 4 = 40 \\ 10 \times 10 = 100 \end{array} \right\}$$

$$f(\cos \alpha + \sin \alpha) = \tan \alpha$$

$$f(\cos \alpha) = ?$$

$$f(\cos \alpha + \sin \alpha) = \tan \alpha \Rightarrow \cos^2 \alpha + \sin^2 \alpha - 2 \cos \alpha \sin \alpha = \tan^2 \alpha \Rightarrow 1 - 2 \sin \alpha \cos \alpha = \tan^2 \alpha \Rightarrow (\cos \alpha + \sin \alpha)(\cos \alpha - \sin \alpha) = \tan^2 \alpha$$

$$\cos \alpha = \frac{1}{\sqrt{2}} \Rightarrow \tan \alpha = \left(\frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} \right) = 1$$

$$\cos \alpha = \frac{1}{\sqrt{2}} \Rightarrow \alpha = \frac{\pi}{4}$$

