

$$D_f = (-\infty, 0) \cup \left[\frac{1}{2}, 1\right)$$

①

الف) $f(x) = \sqrt{\frac{x-1}{x}} - \frac{x}{x-1}$

$$\frac{(x-1) - x^2}{(x)(x-1)} = \frac{x^2 + 1 - 2x - x^2}{x^2 - x} = \frac{1 - 2x}{(x)(x-1)}$$

ب) $f(x) = \frac{1}{x+1} - \frac{2}{x}$

$$\frac{1}{x+1} - \frac{2}{x} = 0 \Rightarrow \frac{x - 2(x+1)}{x(x+1)} = 0 \Rightarrow \frac{x - 2x - 2}{x(x+1)} = 0 \Rightarrow \frac{-x - 2}{x(x+1)} = 0$$

(1) $x - 1 \neq 0 \Rightarrow x \neq 1$

(2) $x + 2 \neq 0 \Rightarrow x \neq -2$

(3) $\frac{3}{x-1} + \frac{1}{x+2} = 0 \Rightarrow \frac{3}{x-1} = -\frac{1}{x+2} \Rightarrow 3x + 4 = -x + 1 \Rightarrow 4x = -3 \Rightarrow x = -\frac{3}{4}$

(4) $x + 1 \neq 0 \Rightarrow x \neq -1$

$$D_f = \mathbb{R} - \left\{-2, -\frac{3}{4}, -1, 0, 1\right\}$$

(5) $x \neq 0$

الف) $f(x) = \sqrt{\left(\frac{1}{x}\right)^2 - 9} (32 - x^2)$

$$\frac{1}{x^2} - 9 \geq 0 \Rightarrow \frac{1 - 9x^2}{x^2} \geq 0 \Rightarrow \frac{(1-3x)(1+3x)}{x^2} \geq 0$$

②

$x < -\frac{1}{3}$ or $x > \frac{1}{3}$

$$D_f = (-\infty, -\frac{1}{3}] \cup [\frac{1}{3}, +\infty)$$

ب) $\sqrt{x-1} + \sqrt{y+1} = 3 \Rightarrow \sqrt{y+1} = 3 - \sqrt{x-1} \Rightarrow 3 \geq \sqrt{x-1} \Rightarrow 9 \geq x-1 \Rightarrow x \leq 10$

(II) $x-1 \geq 0 \Rightarrow x \geq 1$

$D_f = [1, 10]$

③

$f(x) = \log_{\frac{x-x-2}{\sqrt{x-1}+1}}$

مخرج عبارات همواره مثبت است و مخرج خود

① $x^2 - x - 2 > 0 \Rightarrow (x-2)(x+1) > 0 \Rightarrow x < -1, x > 2$

② $x^2 - 1 > 0 \Rightarrow x^2 > 1 \Rightarrow x < -1, x > 1$

① ② $\Rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$



$$f(x) = \sqrt{x^2 + ax - x^2}$$

$$[-2, b]$$

با توجه به دامنه عبارت و می توان گفت (۲) ریشه عبارت
خبر را دیکال می باشد.

$$x = -2 \rightarrow x^2 - 2ax + x^2 = 0 \rightarrow 2a = -1 \Rightarrow a = -\frac{1}{2}$$

$$-x^2 - \frac{1}{2}x + x^2 \xrightarrow{\text{مجموع دلتا}} \frac{1}{2} \pm \sqrt{\frac{1}{4} + 1} \Rightarrow \frac{1}{2} \pm \frac{\sqrt{5}}{2} \rightarrow (-2)$$

$$\frac{\frac{1}{2} + \frac{\sqrt{5}}{2}}{-2} \rightarrow \left(\frac{\frac{1}{2} + \frac{\sqrt{5}}{2}}{-2}\right) = b$$

$$a + b \Rightarrow \left[-\frac{1}{2} + \frac{\sqrt{5}}{2} = 1\right]$$

$$f(x) = \begin{cases} x^2 - 2 & x \geq 1 \\ x^2 + 3 & x < 1 \end{cases}$$

$$g(x) = \sqrt{f(x) - x} \Rightarrow f(x) \geq x \xrightarrow{(1)} x^2 - 2 \geq x \Rightarrow x^2 - x - 2 \Rightarrow [x \geq 1]$$

$$\xrightarrow{\text{انفصال با اعداد}} [x \geq 1]$$

$$\xrightarrow{(2)} x^2 + 3 \geq x \Rightarrow [x \geq -3] \xrightarrow{\text{انفصال با اعداد}} [-3, x < 1]$$

$$(1) \cup (2) \Rightarrow \bigcup_f \Rightarrow [-3, +\infty)$$

$$f(x) = x^2 - x \Rightarrow \frac{x(\sqrt{x^2 + 1})}{x} - \frac{x(\sqrt{x^2 + 1})}{x} = \frac{x + x\sqrt{x^2 + 1}}{x} = \frac{x\sqrt{x^2 + 1} - 1}{x} = \left[-\frac{1}{2}\right]$$

$$f\left(\sin \frac{\pi}{3} + \cos 2\pi\right)$$

$$\Rightarrow \sin \frac{\pi}{3} + \cos 2\pi = \frac{\sqrt{3}}{2} + 1 \Rightarrow \frac{\sqrt{3} + 2}{2}$$

$$f(n) = \frac{n+1}{n-1}$$

$$f(f(a)) = a \Rightarrow f(a) = \frac{a+1}{a-1}$$

مرحله اول:

(V)

$$f(f(a)) \Rightarrow \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{\frac{a+1+a-1}{a-1}}{\frac{a+1-a+1}{a-1}} = \frac{2a}{2} = a$$

پسند:

یعنی $f(f(a))$ برای هر a در دامنه تابع با a برابر است.

$$x \neq 1 \Rightarrow f(f(a)) \neq 1 \Rightarrow a \neq 1$$

$$a \in \mathbb{R}, a \neq 1$$

$$f(x) = x^2 - 5x$$

$$g(x) = \sqrt{x^2 - 1}$$

$$f(g(a)) = -4 \Rightarrow a^2 - 5a = -4 \Rightarrow a^2 - 5a + 4 = 0 \Rightarrow (a-1)(a-4) = 0 \Rightarrow a = 1 \text{ or } a = 4$$

$$g(a) = 2 \Rightarrow \sqrt{a^2 - 1} = 2 \Rightarrow a^2 - 1 = 4 \Rightarrow a^2 = 5 \Rightarrow a = \pm\sqrt{5}$$

$$f(4) \times f(-4)$$

$$f(x+4) = x^2 - 5x + 4$$

$$x+4 = 4 \Rightarrow x = 0 \Rightarrow 1 - 0 + 4 = 5$$

$$x+4 = -4 \Rightarrow x = -8 \Rightarrow 64 - 40 + 4 = 28$$

$$\left. \begin{array}{l} 5 \times 28 = 140 \end{array} \right\}$$

$$f(\cos \alpha + \sin \alpha) = \tan \alpha \quad f(\cos \alpha) = ?$$

$$f(\cos \alpha + \sin \alpha) = \tan \alpha \Rightarrow \cos^2 \alpha + \sin^2 \alpha - 2 \cos \alpha \sin \alpha = \tan^2 \alpha \Rightarrow 1 - 2 \sin \alpha \cos \alpha = \tan^2 \alpha \Rightarrow (\cos \alpha + \sin \alpha)(\cos \alpha - \sin \alpha) = \tan^2 \alpha$$

$$\cos \alpha = \frac{1}{\sqrt{2}} \Rightarrow \sin \alpha = \frac{1}{\sqrt{2}} \Rightarrow \tan \alpha = \left(\frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} \right) = 1 \Rightarrow \alpha = 45^\circ$$

$$g = \frac{1}{\sqrt{2}} \quad \cos \alpha = \frac{1}{\sqrt{2}}$$

