

۲. آفرین های نرسته

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \Rightarrow \sqrt{\frac{(x-1)^2 - x^2}{x(x-1)}} \Rightarrow \sqrt{\frac{(x-1-x)(x-1+x)}{x(x-1)}} \Rightarrow \sqrt{\frac{-1}{x}}$

جدول علامت

0	$\frac{1}{x}$	1
+	-	+
∞	0	∞

$D_f = (-\infty, 0) \cup [\frac{1}{x}, 1)$

ب) $f(x) = \frac{1}{x+1} - \frac{x}{x-1} \Rightarrow \frac{x-1-x(x+1)}{(x+1)(x-1)} \Rightarrow \frac{x-1-x^2-x}{(x+1)(x-1)} \Rightarrow \frac{-x^2-2x-1}{(x+1)(x-1)} \Rightarrow \frac{-(x+1)^2}{(x+1)(x-1)} \Rightarrow \frac{-(x+1)}{(x-1)}$

$D_f = \mathbb{R} - \{0, -1, \frac{1}{2}, \frac{1}{x}\}$

الف) $f(x) = \sqrt{(\frac{1}{x}-9)(3x-5)}$

جدول علامت

-2	$\frac{1}{x}$	0
+	-	+
0	0	0

$(-\infty, -2] \cup [\frac{1}{x}, +\infty)$

ب) $\sqrt{x-1} + \sqrt{y+1} \geq 3 \Rightarrow \sqrt{y+1} \geq 3 - \sqrt{x-1}$

مربع طرفین

$x-1 \geq 0 \Rightarrow x \geq 1$

$y+1 \geq 0 \Rightarrow y \geq -1$

$3 - \sqrt{x-1} \geq 0 \Rightarrow \sqrt{x-1} \leq 3 \Rightarrow x-1 \leq 9 \Rightarrow x \leq 10$

$D_f = [1, 10]$

الف) $f(x) = \frac{\log_e}{\sqrt{x^2-1} - 1}$

جدول علامت

-1	0	1
+	-	+
0	0	0

$D_f = (-\infty, -1) \cup [1, +\infty)$

الف) $f(x) = \sqrt{x+ax-a^2}$

جدول علامت

-1	0	1
+	-	+
0	0	0

$D_f = [-1, 1]$

$$b) f(x) = \begin{cases} x^2 - 1 & x \geq 1 \\ x^2 + 1 & x < 1 \end{cases}$$

$$g(x) = \sqrt{f(x) - x}$$

$$\sim x \geq 1 \Rightarrow x^2 - 1 - x \geq x^2 - 1 \quad x \geq 1 \quad \frac{1}{-1+}$$

$$\sim x < 1 \quad x^2 + 1 - x \geq x^2 + 1 \quad \frac{-1}{-1+} \quad D_f = [-1, +\infty)$$

$$4) f\left(\sin \frac{\pi}{x} + \cos \frac{\pi}{x}\right)$$

$$f(x) = x^2 - \varepsilon x$$

$$x^2(x-1)$$

$$\left(x \sin \frac{\pi}{x}, x \cos \frac{\pi}{x}\right) \left(\sin \frac{\pi}{x} + \cos \frac{\pi}{x} - 1\right)$$

$$\frac{x \sqrt{x}}{x} + 1$$

$$\frac{\sqrt{x}}{x} + 1 - 1$$

$$(\sqrt{x} + 1) \left(\frac{\sqrt{x}}{x} - 1\right) = \frac{x}{x} - \sqrt{x} + \sqrt{x} - 1 = -\frac{1}{x}$$

$$v) f(x) = \frac{x+1}{x-1}$$

$$f(x) = \frac{x+1}{x-1} \Rightarrow f(f(x)) = \frac{\frac{x+1}{x-1} + 1}{\frac{x+1}{x-1} - 1}$$

$$D_f = \mathbb{R} \setminus \{1\}$$

$$x-1 \neq 0 \quad \frac{\frac{x+1}{x-1} + 1}{\frac{x+1}{x-1} - 1}$$

$$A) f(x) = x^2 - \varepsilon x$$

$$g(x) = \sqrt{x^2 - 1}$$

$$f(g(x)) = -\varepsilon$$

$$x^2 - \varepsilon x + \varepsilon = 0 \quad (x \neq 1) \quad x = 1$$

$$g(x) = 1 \Rightarrow \sqrt{x^2 - 1} = 1$$

$$|x^2 - 1| = \varepsilon$$

$$x^2 - 1 = \varepsilon$$

$$x^2 \geq 0 \Rightarrow x \geq \pm \sqrt{\varepsilon}$$

$$9) f(f) \times f(-1)$$

$$\forall x \times x = 1 \times \varepsilon$$

$$f(x+1) = x^2 - \varepsilon x + 1$$

$$x+1 = -1 \Rightarrow x = -2$$

$$f(-1) = 1 + 1 - \varepsilon \geq 1 \quad \forall x$$

$$f(\varepsilon) = 1 - \varepsilon + \varepsilon = 1$$

$$x = \varepsilon$$

$$10) \quad f(\sqrt{5}\cos^2 x + \omega \cos x) = \tan^2 x$$

$$f = \sqrt{5}\omega$$

$$\sqrt{5}\cos^2 x + \omega \cos x = \frac{1}{\sqrt{5}} \quad \swarrow \cos x = t$$

$$\sqrt{5}\cos^2 x + 1 \cdot \cos x - \sqrt{5} = 0 \Rightarrow \sqrt{5}t^2 + 1 \cdot t - \sqrt{5} = 0$$

$$t = \frac{-1 \pm \sqrt{1 + 20}}{2\sqrt{5}} \quad \swarrow b = \frac{1}{\sqrt{5}} \checkmark$$

$$t = -\frac{\sqrt{5}}{5} \times \text{G.O.E.}$$

چون $\cos x$ جي قدر $-\frac{\sqrt{5}}{5}$ يا 1 يا -1 ٿيڻ گهرجي.

$$\tan^2 x = \frac{1 - \cos^2 x}{\cos^2 x} \Rightarrow \frac{1 - \frac{1}{5}}{\frac{1}{5}} = \frac{\frac{4}{5}}{\frac{1}{5}} = 4$$

$$f(\sqrt{5}, \omega) = 4$$