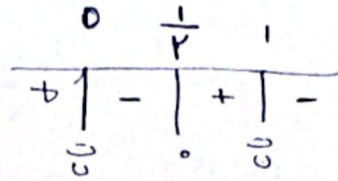


$$1) \text{ ا) } f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \Rightarrow \sqrt{\frac{(x-1)^2 - x^2}{x(x-1)}} \Rightarrow \sqrt{\frac{(x-1-x)(x-1+x)}{x(x-1)}} \quad \text{لأسفل } x$$



$$D_f = (-\infty, 0) \cup \left[\frac{1}{r}, 1\right)$$

$$ب) f(x) = \frac{1}{x+1} - \frac{r}{x} \Rightarrow \frac{x - r(x+1)}{x(x+1)} = \frac{x - rx - r}{x(x+1)} = \frac{-r}{x(x+1)}$$

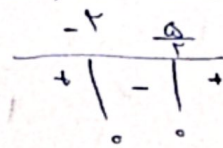
$$\begin{aligned} & \text{as } x \rightarrow -\infty, f(x) \rightarrow 0 \\ & \text{as } x \rightarrow -1, f(x) \rightarrow -\infty \\ & \text{as } x \rightarrow 0, f(x) \rightarrow -\infty \\ & \text{as } x \rightarrow 1, f(x) \rightarrow -\frac{r}{2} \end{aligned}$$

$$D_f = \mathbb{R} - \left\{0, -1, -\frac{a}{r}, \frac{r}{2}\right\}$$

$$2) \text{ ا) } f(x) = \sqrt{\left(\left(\frac{1}{r}\right)^x - 9\right)(r - x^m)}$$

$$\left(\frac{1}{r}\right)^x = 9 \Rightarrow x = -2$$

$$\begin{aligned} r &= 9 \\ r &= r^2 \\ r &= 0 \\ r &= \frac{a}{r} \end{aligned}$$



$$(-\infty, -r] \cup \left[\frac{a}{r}, +\infty\right)$$

$$ب) \sqrt{x-1} + \sqrt{y+1} = r \Rightarrow \sqrt{y+1} = r - \sqrt{x-1}$$

$$\begin{aligned} x-1 &\geq 0 \\ x &\geq 1 \end{aligned}$$

$$\sqrt{y+1} \geq 0$$

$$r \geq \sqrt{x-1}$$

$$r^2 \geq x-1$$

$$r^2 \geq x$$

$$D_f = [1, r^2]$$

$$3) f(x) = \frac{\log_e}{\sqrt{x^2-1} - 1}$$

$$\sqrt{x^2-1} \neq 1$$

$$\begin{aligned} & x^2 - x - r > 0 \\ & (x-r)(x+1) > 0 \end{aligned}$$

$$x^2 - 1 \geq 0$$

$$x^2 \geq 1$$

$$x \geq 1$$

$$x \leq -1$$

$$(x^2 - 1) \geq 0$$

$$D_f = (-\infty, -1] \cup [r, +\infty)$$

$$4) f(x) = \sqrt{r + ax - x^2}$$

$$[-r, b]$$

$$(a-r)(a-b)$$

$$x - r(a-rb) \geq 0$$

$$\begin{aligned} +rb &= r \\ b &= \frac{r}{r} \end{aligned}$$

$$a = +b - r$$

$$a+b = \frac{r}{r} - \frac{1}{r} = 1$$

$$\frac{r}{r} - r = -\frac{1}{r} = a$$

$$b) f(x) = \begin{cases} x^2 - 1 & x \geq 1 \\ x^2 + 1 & x < 1 \end{cases}$$

$$g(x) = \sqrt{f(x) - x}$$

$$\sim, x \geq 1 \Rightarrow x^2 - 1 - x \geq x^2 - 1 \quad x \geq 1 \quad \frac{1}{-1+}$$

$$\sim, x < 1 \quad x^2 + 1 - x \geq x^2 + 1 \quad x < 1 \quad \frac{-1}{-1+}$$

$$D_f = [-1, +\infty)$$

$$4) f\left(\sin \frac{\pi}{4} + \cos \frac{\pi}{4}\right)$$

$$f(x) = x^2 - \varepsilon x$$

$$\left(x \sin \frac{\pi}{4}, x \cos \frac{\pi}{4}\right) \left(\sin \frac{\pi}{4} + \cos \frac{\pi}{4} - 1\right)$$

$$\frac{x \sqrt{2}}{2} + 1$$

$$\frac{\sqrt{2}}{2} + 1 - 1$$

$$\left(\sqrt{2} + 1\right) \left(\frac{\sqrt{2}}{2} - 1\right) = \frac{2}{2} - \sqrt{2} + \sqrt{2} - 1 = -\frac{1}{2}$$

$$v) f(x) = \frac{x+1}{x-1}$$

$$f(x) = \frac{x+1}{x-1}$$

$$f(f(x)) = \frac{\frac{x+1}{x-1} + 1}{\frac{x+1}{x-1} - 1}$$

$$D_f = \mathbb{R} - \{1\}$$

$$x-1 \neq 0 \quad \frac{x+1}{x-1} \neq 1$$

$$\wedge) f(x) = x^2 - \varepsilon x$$

$$g(x) = \sqrt{x^2 - 1}$$

$$f(g(x)) = -\varepsilon$$

$$x^2 - \varepsilon x + \varepsilon = 0 \quad (x \neq 1) \quad x \geq 1$$

$$g(x) = 1$$

$$|x^2 - 1| \geq \varepsilon$$

$$x^2 - 1 \geq \varepsilon \quad x^2 \geq 1 + \varepsilon \quad x \geq \sqrt{1 + \varepsilon}$$

$$9) f(f) \times f(-1)$$

$$\forall x \times x = 1 \quad \varepsilon$$

$$f(x+1) = x^2 - \varepsilon(x+1)$$

$$x+1 \geq -1 \quad x \geq -2$$

$$f(-1) = 1 + 1 - \varepsilon \geq 1 \quad \forall x$$

$$f(\varepsilon) = 1 - \varepsilon + \varepsilon = 1$$

$$x \geq \varepsilon$$

$$10) \quad f(r \cos^2 x + a \cos x) = \tan^2 x$$

$$f, r, a$$

$$r \cos^2 x + a \cos x = \frac{1}{r} \quad \swarrow \cos x = t$$

$$1 \cos^2 x + 1 \cos x - \frac{1}{r} = 0 \Rightarrow 1 t^2 + 1 \cdot t - \frac{1}{r} = 0$$

$$t = \frac{-1 \pm \sqrt{1 + \frac{4}{r}}}{2} \quad \rightarrow \quad b = \frac{1}{r} \checkmark$$

$$t = -\frac{1}{r} \times \text{G.O.E.}$$

چون  $\cos x$  نمی تواند  $-\frac{1}{r}$  باشد  $-1 \leq \cos x \leq 1$  باشد  $\therefore$

$$\tan^2 x = \frac{1 - \cos^2 x}{\cos^2 x} = \frac{1 - \frac{1}{r^2}}{\frac{1}{r^2}} = \frac{r^2 - 1}{1} = 3$$

$$f(3, a) = 3$$