

الف) $\sqrt{\frac{x-1}{x}} - \frac{x}{x-1} \geq 0 \Rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} \geq 0 \Rightarrow x^2 - 2x + 1 - x^2 \geq 0 \Rightarrow -2x + 1 \geq 0 \Rightarrow 2x \leq 1 \Rightarrow x \leq \frac{1}{2}$
 $x \neq 0 \Rightarrow x-1 \neq 0 \Rightarrow x \neq 1$
 $D(f) = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $\frac{1}{x+1} - \frac{1}{x} \geq 0 \Rightarrow \frac{x - (x+1)}{x(x+1)} \geq 0 \Rightarrow \frac{-1}{x(x+1)} \geq 0$
 $\frac{1}{x-1} + \frac{1}{x+1} \geq 0 \Rightarrow \frac{x+1+x-1}{(x-1)(x+1)} \geq 0 \Rightarrow \frac{2x}{(x-1)(x+1)} \geq 0$
 $x \neq 0 \Rightarrow x \neq -1 \Rightarrow x \neq 1$
 $D(f) = \mathbb{R} - \{0, 1, -1, -\frac{1}{2}\}$

الف) $\left(\left(\frac{1}{x}\right)^x - 9\right)(x^2 - x^x) \geq 0$
 $\left(\frac{1}{x}\right)^x - 9 = 0 \Rightarrow \left(\frac{1}{x}\right)^x = 9 \Rightarrow \left(\frac{1}{x}\right)^x = \left(\frac{1}{x}\right)^{-x} \Rightarrow x = -x \Rightarrow x = 0$
 $D(f) = (-\infty, -1] \cup [1, +\infty)$

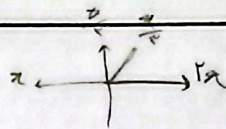
ب) $\sqrt{x-1} + \sqrt{y+1} = 3$
 $\textcircled{1} \rightarrow x-1 \geq 0 \Rightarrow x \geq 1$ $\textcircled{2} \rightarrow y+1 \geq 0 \Rightarrow y \geq -1$
 $A + B = 3 \Rightarrow A \geq 0, B \geq 0 \Rightarrow y+1 = 9 \Rightarrow y = 8$
 $x \in [1, 10], y \in [-1, 8]$

$f(x) = \log_e(x^2 - x - 2)$
 $x^2 - x - 2 > 0 \Rightarrow (x-2)(x+1) > 0 \Rightarrow x < -1 \text{ or } x > 2$
 $\sqrt{x^2-1} + 1 \geq 0 \Rightarrow \sqrt{x^2-1} \geq -1 \Rightarrow x^2-1 \geq 0 \Rightarrow x^2 \geq 1 \Rightarrow x \leq -1 \text{ or } x \geq 1$
 $D(f) = (-\infty, -1) \cup (2, +\infty) \cap (-\infty, -1] \cup [1, +\infty) \Rightarrow (-\infty, -1) \cup (2, +\infty)$

$f(x) = \sqrt{r+ax - x^2}$
 $r + a(-1) - (-1)^2 = 0 \Rightarrow r - a - 1 = 0 \Rightarrow r - a = 1 \Rightarrow a = r - 1$
 $f(x) = \sqrt{-x^2 - \frac{r}{x} + r}$
 $\frac{1}{x} - \frac{f(r)(-1)}{\frac{1}{x^2}} = \sqrt{\frac{f(r)}{r}} \Rightarrow \frac{1}{x} \left[\frac{1}{-1} + \frac{r}{-1} \right] = \sqrt{\frac{f(r)}{r}} \Rightarrow \frac{1}{x} \left[-1 - r \right] = \sqrt{\frac{f(r)}{r}}$

$\sqrt{f(x)-x} \rightarrow f(x)-x \geq 0 \Rightarrow f(x) \geq x$
 $f(x) = \begin{cases} x^2 - x & x \geq 1 \\ x + 2 & x < 1 \end{cases}$
 $\begin{cases} x^2 - x \geq x \rightarrow x^2 \geq 2x \rightarrow x \geq 2 \\ x + 2 \geq x \rightarrow x \geq -2 \end{cases} \Rightarrow D(f) = [-2, 2] \cup [2, +\infty) = [-2, +\infty)$

$$f\left(\frac{\sin x}{\sqrt{r}} + \frac{\cos x}{1}\right)$$



$$f(x) = rx^r - fx$$

$$\Rightarrow f\left(\frac{\sqrt{r+1}}{r}\right)$$

$$r\left(\frac{\sqrt{r+1}}{r}\right)^r - f\left(\frac{\sqrt{r+1}}{r}\right) = \frac{r+1}{r} - \frac{f\sqrt{r+1}}{r} = \frac{r+1-f\sqrt{r+1}}{r} = \frac{r+1-f\sqrt{r+1}}{r}$$

$$f\left(\frac{f(a)}{k}\right) = a$$

$$f(x) = \frac{x+1}{x-1}$$

$$f\left(\frac{a+1}{a-1}\right) = a \Rightarrow \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = a \Rightarrow \frac{a+1+a-1}{a+1-a-1} = a \Rightarrow \frac{2a}{a-2} = a \Rightarrow a-2 = 2 \Rightarrow a = 4$$

$$g(x) = \sqrt{x^r - 1}$$

$$f(x) = x^r - fx$$

$$f(g(a)) = -f \Rightarrow f(k) = -f \Rightarrow k^r - fk = -f \Rightarrow k^r - fk + f = 0 \Rightarrow (k-1)^r = 0 \Rightarrow k = 1$$

$$\Rightarrow g(a) = 1 \Rightarrow \sqrt{a^r - 1} = 1 \Rightarrow a^r - 1 = 1 \Rightarrow a^r = 2 \Rightarrow a = \pm \sqrt[r]{2}$$

$$f(x+r) = x^r - \Delta x + q$$

$$f(r) \times f(-r) = 1 \times 1 = 1$$

$$f(r) \Rightarrow r+r = r \Rightarrow r = 1 \Rightarrow 1 - \Delta + q = 1$$

$$f(-r) \Rightarrow r+r = -r \Rightarrow r = -4 \Rightarrow -4q + r + q = 1$$

$$f(r \cos x + \Delta \cos x) = \tan^r x$$

$$f(r, 0) = ? \quad \cos x = 0, 1 \Rightarrow x = 90^\circ$$

$$\begin{aligned} x = 90^\circ \\ \sin 90^\circ &= \frac{\sqrt{r}}{r} \\ \cos 90^\circ &= \frac{1}{r} \\ \tan 90^\circ &= \sqrt{r} \end{aligned}$$

$$f\left(r \left(\frac{1}{r}\right)^r + \Delta \left(\frac{1}{r}\right)\right) = (\sqrt{r})^r$$

$$f\left(\frac{1}{r} + \frac{\Delta}{r}\right) = \sqrt{r}$$

$$f\left(\frac{1}{r} + \frac{\Delta}{r}\right) = \sqrt{r}$$

$$r \cos^r x + \Delta \cos x = r, 0$$

$$r \cos^r x + \Delta \cos x - r, 0 = 0$$

$$\begin{aligned} -\Delta + \sqrt{r\Delta - f(r)(-r, 0)} &= \frac{-\Delta + 1}{\sqrt{r\Delta - f(r)(-r, 0)}} \\ -\Delta - \sqrt{r\Delta - f(r)(-r, 0)} &= \frac{-\Delta - 1}{\sqrt{r\Delta - f(r)(-r, 0)}} \end{aligned}$$

$$-1 \leq \cos x \leq 1$$