

۲. آزمون عمومی نور

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}}$ $Df = (-\infty, 0) \cup [\frac{1}{2}, 1)$	$x \neq 0, 1$ $\frac{x-1}{x} - \frac{x}{x-1} \geq 0$ $\frac{(x-1)^2 - x^2}{x(x-1)} \geq 0$ $\frac{x^2 - 2x + 1 - x^2}{x(x-1)} \geq 0$ $\frac{1-2x}{x(x-1)} \geq 0$ $\frac{1}{x} - \frac{1}{x-1} \geq 0$	۱
ب) $f(x) = \frac{1}{x+1} - \frac{2}{x}$ $Df = \mathbb{R} - \{-2, -\frac{5}{2}, -1, 0, 1\}$	$x \neq -1, 0, -2, 1$ $\frac{1}{x+1} + \frac{1}{x+2} \neq 0$ $\frac{x+2}{(x+1)(x+2)} \neq 0$ $x+2 \neq 0$ $x \neq -2$ $x \neq -\frac{5}{2}$	۱
الف) $f(x) = \sqrt{((\frac{1}{x})^2 - 9)(32 - 4x)}$ $Df = (-\infty, -2] \cup [\frac{1}{2}, \infty)$	$((\frac{1}{x})^2 - 9)(32 - 4x) \geq 0$ $(\frac{1}{x^2} - 9) \geq 0$ $\frac{1}{x^2} \geq 9$ $x^2 \leq \frac{1}{9}$ $ x \leq \frac{1}{3}$ $x \leq -\frac{1}{3}$ or $x \geq \frac{1}{3}$ $32 - 4x \geq 0 \rightarrow 32 \geq 4x \rightarrow 8 \geq x$ $x \leq 8$	۲
ب) $\sqrt{x-1} + \sqrt{y+1} = 3 \rightarrow \sqrt{y+1} = 3 - \sqrt{x-1}$ $\sqrt{x-1} \geq 0 \rightarrow x \geq 1$ $Df = I \cap II = [1, 82]$	$\sqrt{y+1} \geq 0 \rightarrow 3 - \sqrt{x-1} \geq 0$ $\sqrt{x-1} \leq 3$ $x-1 \leq 9$ $x \leq 10$ $x \leq 10 \cap II$	۲
$f(x) = \frac{\log_e(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$ $Df = (-\infty, -2) \cup (2, \infty)$	$x^2 - x - 2 > 0$ $(x-2)(x+1) > 0$ $x < -1$ or $x > 2$ $x^2 - 1 > 0$ $x^2 > 1 \rightarrow x > 1$ or $x < -1$ II $\sqrt{x^2 - 1} + 1 \neq 0$ $\sqrt{x^2 - 1} \neq -1$ همیشه مثبت	I ۳
$f(x) = \sqrt{3 + ax - x^2}$ $Df = [2, b]$ $a+b=?$ $\frac{1}{2} + \frac{3}{2} = 2$	$3 + ax - x^2 \geq 0$ $x^2 - ax - 3 \leq 0$ $x = b \rightarrow (3 + (\frac{1}{2})xb - b^2 = 0) \times 2$ $-4 + b + 2b^2 = 0$ $\frac{-1 \pm \sqrt{1 - 4 \times 2 \times (-4)}}{2} = \frac{-1 \pm \sqrt{1 + 32}}{4} = \frac{-1 \pm \sqrt{33}}{4}$ $\frac{-1 + \sqrt{33}}{4} = \frac{3}{2}$ $\Rightarrow b = \frac{3}{2}$ $\frac{-1 - \sqrt{33}}{4} = -2$	۴
$f(x) = \begin{cases} 3x-2 & x \geq 1 \\ 2x+3 & x < 1 \end{cases}$ $g(x) = \sqrt{f(x) - x}$ $Dg = [-2, +\infty)$	$f(x) \geq x$ $x \geq 1 \rightarrow 3x-2-x = 2x-2 \geq 0$ $2x-2 \geq 0 \rightarrow x \geq 1$ $x < 1 \rightarrow 2x+3-x = x+3 \geq 0 \Rightarrow x \geq -3$	۵

$$f(x) = x^2 - 5x$$

$$f\left(\underbrace{\sin \frac{\pi}{10}}_{\frac{\sqrt{5}}{2}} + \underbrace{\cos 2\pi}_1\right) = ? \Rightarrow f\left(\frac{\sqrt{5}}{2} + 1\right)$$

$$f\left(1 + \frac{\sqrt{5}}{2}\right) = \left(1 + \frac{\sqrt{5}}{2}\right)^2 - \left(1 + \frac{\sqrt{5}}{2}\right) = \left(\frac{4}{4} + 1 + \frac{\sqrt{5}}{2}\right) - \left(1 + \frac{\sqrt{5}}{2}\right)$$

$$\frac{4}{4} + 1 + \frac{\sqrt{5}}{2} - 1 - \frac{\sqrt{5}}{2} = -1 + \frac{4}{4} = -1 + 1 = 0 \quad \text{D}$$

$$f(x) = \frac{x+1}{x-1}, \quad f(f(a)) = a$$

$$f(a) = \frac{a+1}{a-1}$$

$$f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1}$$

$$\begin{aligned} & \text{تساوی برقرار است} \\ & a-1 \neq 0 \\ & (a \neq 1) \quad \text{D} \end{aligned}$$

$$f(x) = x^2 - 5x$$

$$g(x) = \sqrt{x^2 - 1}$$

$$f(g(a)) = -5$$

$$a = ?$$

$$x^2 - 5x = -5$$

$$f(x) = -5$$

$$x^2 - 5x + 5 = 0$$

$$g(a) = 2$$

$$\sqrt{x^2 - 1} = 2$$

$$(x-5)^2 = 0 \Rightarrow x = 5$$

$$x^2 - 1 = 5$$

$$x^2 = 6$$

$$x = \pm \sqrt{6} \quad \text{D}$$

$$f(x+y) = x^2 \cos y + y$$

$$f(5) \times f(-5) =$$

$$x+y=5 \Rightarrow x=1$$

$$\left(\frac{1-5+y}{1}\right) \times \left(\frac{5+y+4 \times 5}{1}\right) = \frac{(1-5+y)(5+y+20)}{1} = \frac{(y-4)(y+25)}{1}$$

$$\begin{aligned} x+y &= 5 \\ x &= -5 \end{aligned}$$

$$f(5 \cos x + 5 \cos x) = \tan^2 x$$

$$f(5, 5) = ?$$

$$5 \cos^2 x + 5 \cos x = 5, 5$$

$$(5 \cos^2 x + 5 \cos x - 5, 5 = 0) \times 2$$

$$10 \cos^2 x + 10 \cos x - 10 = 0$$

$$t = \frac{-10 \pm \sqrt{10^2 - 4(1)(-10)}}{19} = \frac{-10 \pm \sqrt{300}}{19}$$

$$-1 \leq \cos x \leq 1$$

$$\cos x = \frac{-10 \pm 10}{19}$$

$$\cos x = \frac{1}{19}$$

$$\cos x = \frac{1}{19}$$

$$f(5, 5) = \tan^2 x = (\sqrt{19})^2 = 19$$

$$\frac{1}{\cos x} = \sqrt{19} \Rightarrow \cos x = \frac{1}{\sqrt{19}}$$

$$\cos x = \frac{1}{19} \Rightarrow \cos x = \frac{1}{19}$$