

$$\text{Q1)} f'(x) = \sqrt{\frac{x-1}{x}} - \frac{x}{x-1}$$

$$\textcircled{1} x \neq 0, x \neq 1$$

Prob ①

$$\textcircled{2} \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \Rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} \geq 0 \Rightarrow \frac{-2x+1}{x(x-1)} \geq 0$$

$$\Rightarrow \frac{1}{x(x-1)} \leq 0$$

$$\frac{1}{x(x-1)} \leq 0 \Rightarrow D_f = (-\infty, 0) \cup [1, +\infty)$$

$$\text{b)} f(x) = \frac{\frac{1}{x+1} - \frac{x}{x-1}}{\frac{x}{x-1} + \frac{1}{x+1}}$$

$$x \neq -1, x \neq 0, x \neq 1, x \neq -2$$

$$\frac{x}{x-1} + \frac{1}{x+2} \neq 0 \Rightarrow \frac{x(x+2) + 1(x-1)}{(x-1)(x+2)} \neq 0 \Rightarrow \frac{x^2 + 3x - 1}{(x-1)(x+2)} \neq 0$$

$$x^2 + 3x - 1 \neq 0 \Rightarrow x \neq \frac{-3 \pm \sqrt{13}}{2} \Rightarrow D_f = \mathbb{R} - \left\{ -\frac{3}{2}, -1, -\frac{5}{2}, 0, 1 \right\}$$

$$\text{Q1)} f(x) = \sqrt{\left(\left(\frac{1}{x}\right)^x - 9\right)(3x - x^2)}$$

Prob ②

$$\left(\left(\frac{1}{x}\right)^x - 9\right)(3x - x^2) \geq 0$$

$$\frac{-x}{x-1} \geq 0$$

$$\begin{matrix} \xrightarrow{x < 0} x = -x \\ \xrightarrow{x > 0} x = x, 2 \end{matrix}$$

$$\Rightarrow (-\infty, -1] \cup \left[\frac{2}{3}, +\infty\right)$$

$$\text{b)} \sqrt[3]{x-1} + \sqrt{y+1} = 3$$

$$\textcircled{1} x-1 \geq 0 \Rightarrow x \geq 1$$

$$\textcircled{2} \sqrt[3]{x-1} \leq 3 \Rightarrow x-1 \leq 27 \Rightarrow x \leq 28$$

$$\Rightarrow D_f = [1, 28]$$

$$f(x) = \frac{\log_2(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$$

Prob ③

$$\textcircled{1} x^2 - x - 2 > 0 \Rightarrow x = -1, x = 2 \Rightarrow \frac{-1}{x-1} > 0 \Rightarrow x > 2, x < -1 \textcircled{2} +0, \log_2 x$$

$$\textcircled{3} x^2 - 1 \geq 0 \Rightarrow x = -1, x = 1 \Rightarrow \frac{-1}{x-1} \geq 0 \Rightarrow x \geq 1, x \leq -1 \Rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$$

$$f(x) = \sqrt{x+ax-n^2}, D_f = [-r, b] \Rightarrow a+b=? \quad (5)$$

$$x+ax-n^2 \geq 0 \Rightarrow x^2-ax-n^2 \leq 0 \quad x=-r, x=b$$

$$\frac{-b}{a} = -r+b \Rightarrow \frac{a}{1} = -r+b \quad \frac{c}{a} = b-r \Rightarrow -r = b-r \Rightarrow b = \frac{r}{r}$$

$$\hookrightarrow a = -r + \frac{r}{r} \Rightarrow a = -\frac{1}{r} \Rightarrow a+b = 1$$

$$f(x) = \begin{cases} x^n - r & x \geq 1 \\ rx + r & x < 1 \end{cases}$$

$$\Rightarrow D_g=? \quad g(x) = \sqrt{f(x)-n}$$

$$f(x) \geq n \quad \boxed{I}$$

$$\Rightarrow x^n - r \geq n \Rightarrow x^n - n \geq r \Rightarrow x \geq 1 \quad [1, +\infty)$$

$$\Rightarrow rx + r \geq n \Rightarrow rx - n \geq -r \Rightarrow x \geq -\frac{r}{r} \Rightarrow [-\frac{r}{r}, 1) \quad \boxed{II}$$

$$\Rightarrow D_g = [-\frac{r}{r}, +\infty)$$

$$f(x) = rx^r - \frac{r}{2}x \Rightarrow f\left(\sin \frac{\pi}{r} + \cos r\pi\right) = ? \quad (6)$$

$$\rightarrow f\left(\frac{\sqrt{r}}{r} + 1\right)$$

$$\Rightarrow f\left(\frac{\sqrt{r}}{r} + 1\right) = r\left(\frac{\sqrt{r}}{r} + 1\right)^r - \frac{r}{2}\left(\frac{\sqrt{r}}{r} + 1\right) \Rightarrow \frac{r}{r} + \cancel{r\sqrt{r}} - \cancel{r\sqrt{r}} - \frac{r}{2} = \frac{r}{r} - \frac{r}{2} = -\frac{1}{r}$$

$$f(x) = \frac{n+1}{n-1} \quad f(f(a)) = a \quad (7)$$

$$f(f(x)) = \frac{\frac{n+1}{n-1} + 1}{\frac{n+1}{n-1} - 1} = x$$

$$\hookrightarrow D_{(f)} = \{a \mid a \in \mathbb{Z}, a \neq 1\}$$

$$f(x) = x^r - \varepsilon, \quad g(x) = \sqrt{x^r - 1}, \quad f(g(a)) = -\varepsilon \Rightarrow a = ? \quad (1)$$

$\hookrightarrow \sqrt{a^r - 1}$

$$f(\sqrt{a^r - 1}) = (\sqrt{a^r - 1})^r - \varepsilon \sqrt{a^r - 1} = -\varepsilon \rightarrow K^r - \varepsilon K + \varepsilon = 0$$

$$(K - \varepsilon)^r = 0 \Rightarrow K = \varepsilon \Rightarrow \sqrt{a^r - 1} = \varepsilon \rightarrow a^r - 1 = \varepsilon^2 \Rightarrow a^r = 1 + \varepsilon^2$$

$$a = \pm \sqrt[1/r]{1 + \varepsilon^2}$$

$$f(m + \pi) = m^r - am + \pi \Rightarrow f(\varepsilon) \times f(-\varepsilon) = ? \quad (9)$$

$$f(\varepsilon) \rightarrow m + \pi = \varepsilon \Rightarrow m = 1 \rightarrow f(\varepsilon) = 1 - \varepsilon + \pi = \pi \Rightarrow \pi \times \pi = \pi^2$$

$$f(-\varepsilon) \rightarrow m + \pi = -\varepsilon \Rightarrow m = -1 \rightarrow f(-\varepsilon) = 1 + \varepsilon + \pi = \pi$$

$$f(\varepsilon \cos^r m + \delta \cos m) = \tan^r m \Rightarrow f(\varepsilon, \delta) = ? \quad (10)$$

$$\varepsilon \cos^r m + \delta \cos m = \pi \Rightarrow \cos m = t$$

$$\varepsilon t^r + \delta t = \pi \Rightarrow \Delta t^r + 10t - \pi = 0 \Rightarrow \Delta = 1 \dots \pi^2 \varepsilon = \pi^2 \varepsilon$$

$\sqrt{\pi^2 \varepsilon} = \pi$

$$\rightarrow \frac{t}{1} = \frac{1}{\pi} \quad , \quad \frac{t}{\pi} = \frac{\pi}{\varepsilon}$$

$\frac{1}{\pi} \in [-1, 1] \times$

$$\Rightarrow \cos m = \frac{1}{\pi} \Rightarrow \tan^r m = \frac{\frac{\pi}{\varepsilon}}{\frac{1}{\pi}} = \pi \Rightarrow f(\varepsilon, \delta) = \pi$$