

1. $\frac{1}{x^2} = x^{-2}$

$$f(x) = \int \frac{x-1}{x} - \frac{x}{x-1}$$

$$\frac{(x-1)^2 - x^2}{x(x-1)} \geq 0 \rightarrow \frac{x^2 - 2x + 1 - x^2}{x(x-1)} = \frac{-2x + 1}{x(x-1)}$$

$$\frac{-2x+1}{x(x-1)} \geq 0 \rightarrow D_f = (-\infty, 0) \cup [1, +\infty)$$

$$f(x) = \frac{\frac{1}{x-1} - \frac{x}{x}}{\frac{x}{x-1} + \frac{1}{x+1}} = \frac{\frac{1-x}{x(x-1)}}{\frac{x^2 + x - x - 1}{x(x-1)}} = \frac{1-x}{x^2 + x - 1}$$

$$\frac{-x-v \rightarrow (-x)}{x(x+1)} \rightarrow \frac{(-x)}{x(x+1)} \rightarrow \frac{-1}{x+1}$$

$$D_f = \mathbb{R} - \left\{ -1, -\frac{1}{x}, -1, 0, 1 \right\}$$

2.

$$f(x) = \int \left[\left(\frac{1}{x} \right)^x - q \right] (x^x - x^x)$$

$$\frac{x}{x+1} = \frac{x^x - x^{x-1}}{x^x - x^{x-1} + x^{x-1} - x^{x-2}} = \frac{x^x - x^{x-1}}{x^x - x^{x-2}}$$

$$D_f = (-\infty, -1] \cup [1, +\infty)$$

$$D_f = [1, +\infty)$$

3.

$$f(x) = \frac{\log_k (x^x - x - x)}{\sqrt{x^x - 1} + 1}$$

$$x^x - 1 > 0 \rightarrow x^x - 1 > 0 \rightarrow x^x > 1 \rightarrow x > 1$$

$$f(x) = \begin{cases} x^x - 1 & x \geq 1 \\ x^x + 1 & x < 1 \end{cases}$$

$$f(x) = \int \frac{x^x - x^x}{x^x - 1}$$

$$f(x) = \int \frac{x^x - x^x}{x^x - 1}$$

$$x^x - 1 > 0 \rightarrow x^x > 1 \rightarrow [1, +\infty)$$

$$D_g = (-\infty, -1) \cup [1, +\infty)$$

$$D_g = [-x^x + \infty)$$

5.

$$f(x) = \sqrt{x + x - x^r}$$

$$D_f = (-r, b)$$

$$\frac{1}{-b + \frac{b}{r}}$$

$$\boxed{b = \frac{r}{r-1}} \quad \boxed{a = -\frac{1}{r-1}}$$

$$a + b = \frac{r}{r-1} = \sqrt{r}$$

$$x = -1 \rightarrow r + (-1)^r - (-1)^r = 0$$

$$(-1)^{r-1} = 0 \rightarrow a = -\frac{1}{r}$$

$$x = b \quad r + ab - b^r = 0$$

$$r - \frac{1}{r} b - b^r = 0 \rightarrow b^{r+1} b^{-1/r} = 0 \rightarrow \Delta = \frac{1}{r} - r(1)(r) = \frac{1}{r} + \frac{r}{r} = \frac{r+1}{r}$$

$$\frac{-\frac{1}{r} \pm \frac{r}{r}}{r} \quad \left(\begin{array}{l} b = \frac{r}{r-1} \\ b = (-1) \end{array} \right) \checkmark$$

6

$$f(x) = rx^r - x^r \rightarrow rx^r (x^{r-r}) \Rightarrow (5^{r+1}) \left(\frac{5^{r+1} - 5}{r} \right)$$

$$\left(\frac{5^{r+1} - 5}{r} \right) (5^{r+1}) \Rightarrow \frac{r-5}{r} = \left(-\frac{1}{r} \right)$$

$$\frac{5^r}{r} \quad \frac{r}{r}$$

7

$$f\left(\sin \frac{\pi}{r} + \cos \frac{\pi}{r}\right) = g \quad f\left(\frac{5^{r+1}}{r}\right) = ?$$

$$f(x) = \frac{x+1}{x-1}$$

$$f(a) = \frac{a+1}{a-1}$$

$$f(f(a)) =$$

$$\frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} \Rightarrow \frac{\frac{a+1 + a-1}{a-1}}{\frac{a+1 - (a-1)}{a-1}} = \frac{r_a}{r_a} = \frac{r_a}{r_a} = 1$$

$$f(f(a)) = a$$

$$a \in \mathbb{R} - \{1\}$$

اذا جازي في اعداد حقيقية

بما ان

8

$$f(x) = x^r - x^r$$

$$f(g(a)) = -r$$

$$a \geq 0$$

$$f(g(a)) = (g(a))^r - r g(a) \Rightarrow \left(\sqrt{a^{r-1}}\right)^r - r \left(\sqrt{a^{r-1}}\right) = -r$$

$$a^{r-1} - r \sqrt{a^{r-1}} = -r$$

$$r = \sqrt{a^{r-1}}$$

$$r^r - r^r + r^r = 0$$

$$(r-r)^r = 0$$

$$\sqrt{a^{r-1}} = r$$

$$(r=r)$$

$$a^{r-1} = r$$

$$a^r = 0 \rightarrow$$

$$a = \pm \sqrt{a}$$

$$a^{r-1} \geq 0$$

اذا جازي في اعداد حقيقية

$$f(x+1) = x^2 - 2x + 4 \quad \times \quad f(x) \times f(-1) = 0 \quad \forall x, 1 = (1 \times 1) \quad \text{بجواب}$$

9

$$f(x+1) \Rightarrow ((x+1)-1)^2 + (x+1) - 4 \Rightarrow f(x) = (x-1)^2 + x - 4$$

$$x^2 + 1 - 2x + x + 1 - 4 = x^2 - x - 2$$

$$f(x) = (-1)^2 + -1 - 4 = -4$$

$$f(-1) = (-1)^2 - 1 - 4 = -4 = \boxed{-4}$$

40

$$f(r \cos \alpha + w \sin \alpha) = \tan \alpha$$

$$f\left(\frac{1}{r}\right) = ?$$

$$\frac{1}{r}$$

$$\boxed{r}$$

$$r \cos \alpha + w \sin \alpha = \frac{\sin \alpha}{\cos \alpha}$$

$$r \cos \alpha + w \sin \alpha = \frac{1}{r}$$

$$\cos \alpha = \frac{1}{r} \Rightarrow r \cos \alpha + w \sin \alpha = \frac{1}{r} \Rightarrow 1 \cos \alpha + 1 \sin \alpha = \frac{1}{r} \Rightarrow \frac{1}{r} =$$

$$\frac{1 \pm \sqrt{1 - 4 \cos \alpha (-\sin \alpha)}}{2 \cos \alpha} \rightarrow \left(\frac{1}{r}\right) \checkmark$$

$$\rightarrow \left(-\frac{1}{r}\right) \times$$

$$\text{if } \cos \alpha = \frac{1}{r} \Rightarrow r \cos \alpha + w \sin \alpha = \frac{1}{r}$$

$$-1 < \cos \alpha = \frac{1}{r} < 1$$

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha} \quad 1 - \cos \alpha \rightarrow \frac{1 - \left(\frac{1}{r}\right)^2}{\left(\frac{1}{r}\right)^2} = \boxed{r}$$

$$\boxed{f\left(\frac{1}{r}\right) = r}$$