

۲. آفرین عابدی دوست من

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \geq 0 \rightarrow \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} \geq 0$

$\frac{1-2x}{x(x-1)} \geq 0 \rightarrow \frac{0}{+} - \frac{1}{+} \rightarrow D = (-\infty, 0) \cup [\frac{1}{2}, 1)$

$2x+4+x-1$

ب) $\frac{1}{x+1} - \frac{2}{x} \rightarrow x \neq -1, 0$ $\frac{3}{x-1} + \frac{1}{x+2} \neq 0 \rightarrow \frac{3(x+2) + (x-1)}{(x-1)(x+2)} \neq 0$ $D = \mathbb{R} - \{-1, -\frac{5}{4}, 0, 1\}$
 $\frac{3}{x-1} + \frac{1}{x+2} \rightarrow x \neq 1, -2$ $\frac{4x+5}{(x-1)(x+2)} \neq 0 \rightarrow 4x+5 \neq 0 \rightarrow x \neq -\frac{5}{4}$

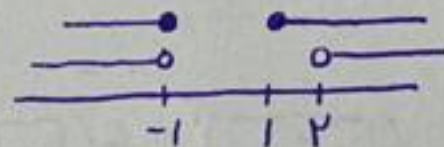
الف) $\sqrt{((\frac{1}{3})^x - 9)(3^x - 4^x)} \rightarrow \frac{-2}{+} - \frac{\frac{5}{3}}{+} \rightarrow D_f = (-\infty, -2] \cup [\frac{5}{3}, +\infty)$

ب) $\sqrt[4]{x-1} + \sqrt{y+1} = 3 \rightarrow \sqrt{y+1} = 3 - \sqrt[4]{x-1} \rightarrow 3 - \sqrt[4]{x-1} \geq 0 \rightarrow \sqrt[4]{x-1} \leq 3 \rightarrow x-1 \leq 81$
 $x-1 \geq 0 \rightarrow x \geq 1$ I $\rightarrow$ عددی نامنفی $x \leq 82$ II

$I \cap II \rightarrow D = [1, 82]$

$f(x) = \frac{\log_4(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1} \rightarrow x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0 \rightarrow x < -1, x > 2$ I

$\rightarrow x^2 - 1 \geq 0 \rightarrow x^2 \geq 1 \rightarrow x \leq -1, x \geq 1$ II



$\sqrt{x^2 - 1} + 1 \neq 0 \rightarrow$ همیشه مثبت و منفی شود چون رادیکال عددی نامنفی است و به علاوه یک مثبت می شود. $I \cap II \rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$

$f(x) = \sqrt{3+ax-x^2} \rightarrow -x^2+ax+3 \xrightarrow{x-1} x^2-ax-3$
 $\frac{c}{a} = -3 = -\frac{1}{b} \rightarrow b = \frac{3}{4}$

جمع ریشه ها: $-\frac{b}{a} = a = -2 + \frac{3}{4} = -\frac{1}{4} \rightarrow a = -\frac{1}{4}$

$a+b = -\frac{1}{4} + \frac{3}{4} = \frac{2}{4} = \frac{1}{2}$

$f(x) = \begin{cases} 3x-2 & x \geq 1 \\ 2x+3 & x < 1 \end{cases}$

$D = [-3, +\infty)$

$g(x) = \sqrt{f(x)-x} \xrightarrow{x \geq 1} \sqrt{(3x-2)-x} = \sqrt{2x-2} \rightarrow 2x-2 \geq 0 \rightarrow x \geq 1$ I

$x < 1 \rightarrow \sqrt{(2x+3)-x} \rightarrow g(x) = \sqrt{x+3} \rightarrow x+3 \geq 0 \rightarrow x \geq -3$

خودش شرط $x < 1$ دارد $\rightarrow [-3, 1)$ II

$$f(x) = 2x^2 - 4x \quad | \quad 1 + \sqrt{3} + \frac{3}{4} = \frac{\sqrt{3}}{4} + \sqrt{3}$$

$$f(\sin \frac{\pi}{3} + \cos 2\pi) \Rightarrow f(1 + \frac{\sqrt{3}}{2}) = 2(1 + \frac{\sqrt{3}}{2})^2 - 4(1 + \frac{\sqrt{3}}{2}) \Rightarrow (\frac{\sqrt{3}}{2} + 2\sqrt{3}) - (4 + 2\sqrt{3})$$

$$\frac{\sqrt{3}}{2} - 4 = -\frac{1}{2} \rightarrow \text{جواب نهایی: } \boxed{-\frac{1}{2}} \quad \text{D}$$

$$f(x) = \frac{x+1}{x-1}$$

$$f(f(a)) = a$$

$$f(a) = \frac{a+1}{a-1} \rightarrow a \neq 1$$

$$D = \mathbb{R} - \{1\}$$

$$f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{\frac{2a}{a-1}}{\frac{2}{a-1}} = a \quad \frac{a+1}{a-1} \neq 1 \rightarrow a+1 = a-1$$

تعدد بیت نداریم $\rightarrow$ غیر ممکن $1 = -1$

$$f(x) = x^2 - 4x$$

$$g(x) = \sqrt{x^2 - 1} \rightarrow g(a) = \sqrt{a^2 - 1}$$

$$f(g(a)) = -4$$

$$f(g(a)) = f(\sqrt{a^2 - 1}) = (\sqrt{a^2 - 1})^2 - 4(\sqrt{a^2 - 1}) = -4 \rightarrow a^2 - 1 - 4\sqrt{a^2 - 1} + 4 = 0 \rightarrow a^2 + 3\sqrt{a^2 - 1} = 0$$

$$a^2 - 1 \leftarrow 4\sqrt{a^2 - 1} \leftarrow t = \sqrt{a^2 - 1}, t^2 = a^2 - 1 \rightarrow t^2 - 4t = -4$$

$$t^2 - 4t + 4 = 0 \rightarrow (t-2)^2 = 0 \rightarrow t = 2 \rightarrow \sqrt{a^2 - 1} = 2$$

$$a^2 - 1 = 4 \rightarrow a^2 = 5 \rightarrow \begin{cases} a = \sqrt{5} \\ a = -\sqrt{5} \end{cases}$$

$$f(x+3) = x^2 - 5x + 4$$

$$f(4) \times f(-3) =$$

$$x=1 \rightarrow f(4) = 1 - 5 + 4 = 0$$

$$x=-4 \rightarrow f(-3) = 9 + 12 + 4 = 25$$

$$f(4) \times f(-3) = 0 \times 25 = 0 \quad \text{D}$$

$$f(4\cos^2 x + 5\cos x) = \tan^2 x$$

$$f(3/5) = ? \rightarrow 4\cos^2 x + 5\cos x = 3/5 \xrightarrow{\cos x = t} 4t^2 + 5t = 3/5 \xrightarrow{\times 5} 20t^2 + 25t = 3$$

$$20t^2 + 25t - 3 = 0 \rightarrow (4t-1)(5t+3) = 0 \rightarrow t = \frac{1}{4}, -\frac{3}{5} \rightarrow -1 \leq \cos x \leq 1 \rightarrow t = \frac{1}{4}$$

$$\sin^2 x + \cos^2 x = 1 \rightarrow \sin^2 x = 1 - \frac{1}{16} = \frac{15}{16} \rightarrow \tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{15}{16}}{\frac{1}{16}} = 15 \quad \text{D}$$