

الف)  $f(x) = \sqrt{\frac{x-1}{x}} - \frac{x}{x-1} \geq 0 \rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} \geq 0 \rightarrow \frac{x^2 - 2x + 1 - x^2}{x(x-1)} \geq 0 \rightarrow \frac{-2x+1}{x(x-1)} \geq 0$   
 $x=0,1$   
 $D_f = (-\infty, 0) \cup (\frac{1}{2}, 1)$

ب)  $\frac{1}{x+1} - \frac{x}{x-1}$   
 $\frac{x}{x-1} + \frac{1}{x+1}$   
 $D_f = \mathbb{R} - \{-1, -\frac{1}{2}, -\frac{1}{x}, 0, 1\}$

الف)  $f(x) = \sqrt{\left(\frac{1}{x}\right)^2 - 9} (x - x^2) \geq 0$   
 $D_f = (-\infty, -2] \cup [\frac{2}{9}, +\infty)$

ب)  $\sqrt{x-1} + \sqrt{y+1} = 3$   
 $x \in [1, 16]$   
 $y \in [-1, 8]$

$f(x) = \frac{\log_2(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$   
 $D_f = (-\infty, -1) \cup (2, +\infty)$

$f(x) = \sqrt{4 + ax - x^2} \geq 0$   
 $D_f = [-2, b]$   
 $a+b = -\frac{1}{2} + \frac{3}{2} = \frac{2}{2} = 1$

$f(x) = \begin{cases} x^2 - 2 & x \geq 1 \\ x^2 + 3 & x < 1 \end{cases}$   
 $g(x) = \sqrt{f(x) - x} \geq 0 \rightarrow f(x) - x \geq 0 \rightarrow f(x) \geq x$   
 $D_g = [-3, +\infty)$

$$P(n) = r^n - kn$$

$$P\left(\sin \frac{\pi}{r} + \cos \frac{\pi}{r}\right) =$$

$$n = \frac{\sin \frac{\pi}{r} + \cos \frac{\pi}{r}}{\frac{\sqrt{r}}{r}} \rightarrow x = \frac{\sqrt{r}}{r} + 1$$

$$\begin{aligned} P(n) &= r\left(\frac{\sqrt{r}}{r} + 1\right)^r - k\left(\frac{\sqrt{r}}{r} + 1\right) = \frac{r}{r} + r\sqrt{r} - r\sqrt{r} - k = \frac{-1}{r} \\ \left(\frac{\sqrt{r}}{r}\right)^r + r\left(\frac{\sqrt{r}}{r}\right) + 1 &= \frac{r}{r} + \sqrt{r} + 1 \\ &= \frac{r}{r} + \frac{r}{r} + \sqrt{r} = \frac{2r}{r} + \sqrt{r} \end{aligned}$$

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$$P(n) = \frac{n+1}{n-1}$$

$$P(P(a)) = a$$

$$a \in \mathbb{R} - \{1\}$$

$$P(a) = \frac{a+1}{a-1}$$

$$\text{نصف الوحدتين} \Rightarrow a \neq 0, a \neq 1$$

$$P(P(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{\frac{2a}{a-1}}{\frac{2}{a-1}} = a$$

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$$P(n) = n^r - kn$$

$$g(a) = \sqrt{a^2 - 1}$$

$$g(n) = \sqrt{n^2 - 1}$$

$$P(g(a)) = (\sqrt{a^2 - 1})^r - k\sqrt{a^2 - 1} = a^r - 1 - k\sqrt{a^2 - 1} = -k$$

$$P(g(a)) = -k$$

$$\rightarrow a^r - 1 - k\sqrt{a^2 - 1} = -k \rightarrow (a^r + k^2)(k\sqrt{a^2 - 1})^r$$

$$a = 9$$

$$\rightarrow (a^r + k^2)^r = 14(a^r - 1) \rightarrow a^r + 9a^r + 9 = 14a^r - 14$$

$$\rightarrow a^r - 10a^r + 20 = 0 \rightarrow (a^r - 5)^r = 0 \rightarrow a^r = 5 \rightarrow a = \pm\sqrt{5}$$

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$$P(n+p) = n^r - an + q$$

$$P(k) \times P(-p) = ? \Rightarrow \forall r, k = 1, k \neq$$

$$\rightarrow n+p = -p \rightarrow n = -4$$

$$P(-4) = (-4)^r - a(-4) + q = 16 + 4a + q = 17$$

$$\rightarrow n+p = 1 \rightarrow n = 1$$

$$P(1) = 1^r - a(1) + q = 1 - a + q = 1$$

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$$P(t \cos^n + a \cos^n) = \tan^n$$

$$P\left(\frac{r}{r}, \frac{a}{r}\right) = ?$$

$$t \cos^n + a \cos^n = \frac{r}{r} \rightarrow (t \cos^n + a \cos^n - \frac{r}{r} = 0) \rightarrow 1t^r + 1at - r = 0$$

$$t a^r = \frac{1}{t^r} - 1 = \frac{1}{(\frac{1}{t})^r} - 1 = \frac{1}{\frac{1}{t^r}} - 1 = t^r - 1 = \frac{r}{r}$$

$$t = \frac{-10 \pm \sqrt{100 + 12t}}{14} = \frac{-10 \pm \sqrt{12t}}{14}$$

$$= \frac{-10 \pm 11}{14} \rightarrow t = \frac{1}{14} = \frac{1}{r} \checkmark$$

$$\sqrt{12t} = \frac{r}{r} = \frac{r}{r} \checkmark$$

$$n \cos \rightarrow -10 \text{ وحدة}$$

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