

بسم الله الرحمن الرحيم !

① $a \neq 0, a \neq 1$
 $f(n) = \sqrt{\frac{n-1}{n} - \frac{n}{n-1}}$
 $\frac{n-1}{n} - \frac{n}{n-1} \geq 0 \Rightarrow \frac{(n-1)^2 - n^2}{n(n-1)} \geq 0 \Rightarrow -\frac{n+1}{n} \geq 0$
 $\frac{1}{-n-1} \geq 0 \Rightarrow D_f = (-\infty, 0) \cup \left(\frac{1}{2}, 1\right)$

② $n \neq -1, 0, 1, -2$
 $f(n) = \frac{\frac{1}{n+1} - \frac{1}{n}}{\frac{1}{n-1} + \frac{1}{n+2}}$
 $\frac{1}{n-1} + \frac{1}{n+2} \neq 0 \Rightarrow \frac{n+2}{(n-1)(n+2)} + \frac{n-1}{(n-1)(n+2)} \neq 0$
 $\frac{2n+1}{(n-1)(n+2)} \neq 0 \Rightarrow D_f = \mathbb{R} - \left\{-\frac{1}{2}, -1, 0, 1\right\}$

③ $f(n) = \sqrt{\left(\frac{1}{n} - 1\right)(2 - \frac{1}{n})} \geq 0 \Rightarrow \left(\frac{1}{n} - 1\right)(2 - \frac{1}{n}) \geq 0$
 $\frac{-2}{n} + \frac{2}{n} - \frac{1}{n} + \frac{1}{n} \geq 0 \Rightarrow D_f = [-\infty, -2] \cup [1, +\infty)$
 $\sqrt{a-1} + \sqrt{a+1} = 2 \Rightarrow \sqrt{a+1} = 2 - \sqrt{a-1}$
 $(2 - \sqrt{a-1})^2 = a+1 \Rightarrow 4 - 4\sqrt{a-1} + a - 1 = a + 1 \Rightarrow -4\sqrt{a-1} = 2 \Rightarrow \sqrt{a-1} = -\frac{1}{2}$
 $a-1 \geq 0 \Rightarrow a \geq 1$
 $D_f = [1, 1]$

④ $f(n) = \log_2 \left(\frac{n(n-1)}{n-1}\right)$
 $\frac{n(n-1)}{n-1} > 0 \Rightarrow n > 1$
 $\frac{n(n-1)}{n-1} = 1 \Rightarrow n = 1$
 $D_f = (-\infty, 1) \cup (1, +\infty)$

⑤ $f(n) = \sqrt{3+an-n^2}, D_f = [-2, b] \rightarrow a+b=?$
 $3+an-n^2 \geq 0 \rightarrow n^2 - an - 3 \leq 0 \rightarrow n = -2, n = b$
 $-\frac{b}{a} = -2+b \rightarrow a = -1+b$
 $a = -2 + \frac{b}{-1+b} \rightarrow a = -\frac{1}{-1+b} \rightarrow a+b = 1$

⑥ $f(n) = \begin{cases} 2n-2 & n \geq 1 \\ 2n+2 & n < 1 \end{cases}$
 $D_f? \quad g(n) = \sqrt{f(n)-n}$
 $f(n) \geq n$
 $2n-2 \geq n \rightarrow n \geq 2 \rightarrow [2, +\infty)$
 $2n+2 \geq n \rightarrow n \geq -2 \rightarrow [-2, 1)$
 $D_f = [-2, 1) \cup [2, +\infty)$

⑦ $f(n) = n^2 - 2n \rightarrow f\left(\sin \frac{\pi}{6} + \cos \frac{\pi}{6}\right) = ? \rightarrow f\left(\frac{\sqrt{3}}{2} + 1\right)?$
 $f\left(\frac{\sqrt{3}}{2} + 1\right) = \left(\frac{\sqrt{3}}{2} + 1\right)^2 - 2\left(\frac{\sqrt{3}}{2} + 1\right) = \frac{3}{4} + \sqrt{3} + 1 - \sqrt{3} - 2 = \frac{3}{4} - 1 = -\frac{1}{4}$

⑧ $f(n) = \frac{n+1}{n-1}, f(f(n)) = a$
 $f(f(n)) = n \rightarrow \frac{\frac{n+1}{n-1} + 1}{\frac{n+1}{n-1} - 1} = n$
 $\frac{n+1}{n-1} - 1 = \frac{n+1 - (n-1)}{n-1} = \frac{2}{n-1}$
 $\frac{n+1}{n-1} + 1 = \frac{n+1 + (n-1)}{n-1} = \frac{2n}{n-1}$
 $D_f = \{a \mid a \in \mathbb{Z}, a \neq 1\}$

$$\begin{cases} f(m) = m^2 - \varepsilon m \\ g(m) = \sqrt{m^2 - 1} \end{cases}$$

$$f(g(a)) = -\varepsilon \rightarrow f(\sqrt{a^2 - 1}) = (\sqrt{a^2 - 1})^2 - \varepsilon \sqrt{a^2 - 1} = -\varepsilon$$

$$\xrightarrow{\sqrt{a^2 - 1}} t^2 - \varepsilon t = -\varepsilon \rightarrow (t - \varepsilon)^2 = 0$$

$$t = \varepsilon \rightarrow \sqrt{a^2 - 1} = \varepsilon$$

$$a = \pm \sqrt{a^2 - 1} \leftarrow \begin{matrix} a^2 - 1 = \varepsilon \\ a^2 = \varepsilon + 1 \end{matrix}$$

$$\begin{aligned} &= -\varepsilon \rightarrow m = -\varepsilon \rightarrow 4 + 0 + 4 = 16 \\ &f(m + c) = m^2 - am + 4 \left\{ \begin{array}{l} f(\varepsilon) = 2 \\ f(-\varepsilon) = 16 \end{array} \right. \quad \text{XVR} \Rightarrow 12\varepsilon \\ &= \varepsilon \\ &\downarrow \\ &m = 1 \rightarrow 1 - a + 4 = 2 \end{aligned}$$

$$\varepsilon \cos^2 m + 0 \cos m - \frac{1}{\varepsilon} = 0$$

$$\cos^2 m + 0 \cos m - 1\varepsilon = 0 \rightarrow (\cos m + \varepsilon)(\cos m - \varepsilon) = 0$$

$$\frac{-\varepsilon}{\varepsilon} \quad \frac{\varepsilon}{\varepsilon} = \left(\frac{1}{\varepsilon}\right)$$

$$\cos = \frac{1}{\varepsilon} \rightarrow \tan^2 m = \frac{\sin^2 m}{\cos^2 m} = \frac{\frac{\varepsilon^2}{\varepsilon^2}}{\frac{1}{\varepsilon^2}} = \varepsilon^2 \rightarrow f(m, \varepsilon) = \varepsilon$$

$$\sin = \frac{\sqrt{\varepsilon}}{\varepsilon}$$

A جزیره دریا