

بسم الله الرحمن الرحيم !

① $a \neq 0, a \neq 1$

$$f(n) = \sqrt{\frac{n-1}{a} - \frac{n}{a-1}}$$

$$\frac{n-1}{a} - \frac{n}{a-1} \geq 0 \Rightarrow \frac{(n-1)^2 - n^2}{a(n-1)} \geq 0 \Rightarrow -\frac{n}{a(n-1)} \geq 0$$

$$\frac{1}{-a} + \frac{1}{a-1} \geq 0 \Rightarrow D_f = (-\infty, 0) \cup \left[\frac{1}{a}, 1\right)$$

② $a \neq -1, a \neq 1, a \neq -2$

$$f(n) = \frac{\frac{1}{n+1} - \frac{1}{n}}{\frac{1}{n-1} + \frac{1}{n+2}}$$

$$\frac{1}{n-1} + \frac{1}{n+2} \neq 0 \Rightarrow \frac{1}{n-1} + \frac{1}{n+2} \neq 0 \Rightarrow \frac{n(n+2) + (n-1)}{(n-1)(n+2)} \neq 0$$

$$\Rightarrow 2n+1 \neq 0 \Rightarrow n \neq -\frac{1}{2} \Rightarrow D_f = \mathbb{R} - \left\{-1, -\frac{1}{2}, -\frac{3}{2}\right\}$$

③ $f(n) = \sqrt{\left(\frac{1}{r}n - 1\right)(r - \frac{1}{r}n)} \geq 0 \Rightarrow \left(\frac{1}{r}n - 1\right)(r - \frac{1}{r}n) \geq 0$

$$\frac{-r}{+} + \frac{r}{-} \geq 0 \Rightarrow D_f = (-\infty, -r] \cup [r, +\infty)$$

④ $\sqrt{a-1} + \sqrt{a+1} = r \Rightarrow \sqrt{a+1} = r - \sqrt{a-1}$

$$\sqrt{a+1} \geq 0 \Rightarrow r - \sqrt{a-1} \geq 0 \Rightarrow \sqrt{a-1} \leq r \Rightarrow a-1 \leq r^2 \Rightarrow a \leq r^2 + 1$$

$$D_f = [1, r^2 + 1]$$

⑤ $f(n) = \log_2 \left(\frac{n(n-1)}{n^2 - 1} + 1\right)$

⑥ $a^2 - 1 \geq 0 \Rightarrow a^2 \geq 1 \Rightarrow a \geq 1, a \leq -1$

⑦ $a^2 - 1 \geq 0 \Rightarrow a \geq 1, a \leq -1$

⑧ $D_f = (-\infty, -1) \cup (1, +\infty)$

⑨ $f(n) = \sqrt{r+an-n^2}, D_f = [-r, b] \rightarrow a+b=?$

$r+an-n^2 \geq 0 \Rightarrow n^2 - an - r \leq 0 \Rightarrow n = -r, n = b$

$-\frac{b}{a} = -r+b \Rightarrow a = -b$ $\frac{c}{a} = b+1-r \Rightarrow -c = b+1-r$

$a = -r + \frac{c}{r} \rightarrow a = -\frac{1}{r} \rightarrow a+b = 1$ $b = \frac{r}{r}$

⑩ $f(n) = \begin{cases} r_n - r & n \geq 1 \\ r_n + c & n < 1 \end{cases}$

Deg? $g(n) = \sqrt{f(n) - n}$

$f(n) \geq n$

$\Rightarrow r_n - r \geq n \Rightarrow r_n - n \geq r \Rightarrow n \geq 1 \Rightarrow [1, +\infty)$

$\Rightarrow r_n + c \geq n \Rightarrow r_n - n \geq -c \Rightarrow n \geq -c \Rightarrow [-c, 1)$

$D_f = [-c, +\infty)$

⑪ $f(n) = r_n^2 - \frac{1}{r}n \rightarrow f\left(\sin \frac{\pi}{r} + \cos r\pi\right) = ? \rightarrow f\left(\sqrt{\frac{r}{r}} + 1\right) = ?$

$f\left(\sqrt{\frac{r}{r}} + 1\right) = r\left(\sqrt{\frac{r}{r}} + 1\right)^2 - \frac{1}{r}\left(\sqrt{\frac{r}{r}} + 1\right) \Rightarrow \frac{r}{r} + \frac{r}{r} + r - \frac{1}{r} - \frac{1}{r} = r$

⑫ $D_f = \{a \mid a \in \mathbb{Z}, a \neq 1\}$

⑬ $f(n) = \frac{n+1}{n-1}, f(f(n)) = a$

$f(f(n)) = a \rightarrow \frac{\frac{n+1}{n-1} + 1}{\frac{n+1}{n-1} - 1} = a$

⑭ $D_f = \{a \mid a \in \mathbb{Z}, a \neq 1\}$

$$\begin{cases} f(m) = m^2 - \varepsilon m \\ g(m) = \sqrt{m^2 - 1} \end{cases}$$

$$f(g(a)) = -\varepsilon \rightarrow f(\sqrt{a^2 - 1}) = (\sqrt{a^2 - 1})^2 - \varepsilon \sqrt{a^2 - 1} = -\varepsilon$$

$$\xrightarrow{\sqrt{a^2 - 1}} t^2 - \varepsilon t = -\varepsilon \rightarrow (t - \varepsilon)^2 = 0$$

$$t = \varepsilon \rightarrow \sqrt{a^2 - 1} = \varepsilon$$

$$a^2 - 1 = \varepsilon$$

$$a = \pm \sqrt{\omega} \leftarrow a^2 = \omega$$

$$\begin{aligned} &= -\varepsilon \rightarrow m = -\varepsilon \rightarrow 4 + 0 + 4 = 16 \\ &f(m + c) = m^2 - am + 4 \left\{ \begin{array}{l} f(\varepsilon) = 2 \\ f(-\varepsilon) = 16 \end{array} \right\} \text{XVR} \Rightarrow 122 \end{aligned}$$

$$\downarrow$$

$$m = 1 \rightarrow 1 - a + 4 = 2$$

$$\varepsilon \cos^2 m + 0 \cos m - \frac{1}{\varepsilon} = 0$$

$$\cos^2 m + 0 \cos m - 1\varepsilon = 0 \rightarrow (\cos m + \varepsilon)(\cos m - \varepsilon) = 0$$

$$\frac{-\varepsilon}{\varepsilon} \quad \frac{\varepsilon}{\varepsilon} = \left(\frac{1}{\varepsilon}\right)$$

$$\cos = \frac{1}{\varepsilon} \rightarrow \tan^2 m = \frac{\sin^2 m}{\cos^2 m} = \frac{\frac{\sqrt{1 - \frac{1}{\varepsilon^2}}}{\frac{1}{\varepsilon}}}{\frac{1}{\varepsilon^2}} = \varepsilon \rightarrow f(m, 0) = \varepsilon$$

$$\sin = \left(\frac{\sqrt{1 - \frac{1}{\varepsilon^2}}}{\varepsilon}\right)$$

A جزیره دریا