

الف)  $\sqrt{\frac{x^2 - 2x + 1 - x^2}{x(x-1)}} = \sqrt{\frac{-2x+1}{x(x-1)}}$   $\frac{0}{+ \quad - \quad - \quad +}$   $D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب)  $\frac{x^2 - 5x - 2}{(x+1)x}$   $x \neq 0, -1, 1, -2$   $D_f = R - \{\pm 1, 0, -2, -\frac{5}{4}\}$   
 $\frac{x^2 + 9 + x - 1}{(x-1)(x+2)} \rightarrow 4x + 5 \neq 0 \quad x \neq -\frac{5}{4}$

الف)  $(\frac{1}{x})^n - 9)(32 - x^n) \geq 0$   $x = -2 \rightarrow$   $\frac{-2}{+ \quad - \quad +}$   $D_f = (-\infty, -2] \cup [\frac{5}{2}, +\infty)$   
 $x = \frac{5}{2} \rightarrow$

ب)  $\sqrt{x-1} + \sqrt{y+1} = 3$   $\textcircled{1} x \geq 1$   
 $\textcircled{2} \sqrt{x-1} \leq 3 \quad x-1 \leq 9 \quad x \leq 10$   $1 \leq x \leq 10 \Rightarrow D_f = [1, 10]$

$\textcircled{1} x^2 - x - 2 > 0 \quad (x-2)(x+1) > 0$   $\frac{-1}{+ \quad - \quad +}$   $\rightarrow x > 2 \cup x < -1$

$\textcircled{2} x^2 - 1 \geq 0 \quad (x-1)(x+1) \geq 0$   $\frac{-1}{+ \quad - \quad +}$   $\rightarrow x \geq 1 \cup x \leq -1$

$\textcircled{3}$  معین همواره برتر از صفر

$\textcircled{1} \cap \textcircled{2} \cap \textcircled{3} \Rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$

$x = -2 \rightarrow$   $\sqrt{r+ax-x^2} \rightarrow +3 - 2a - 4 = 0 \quad a = -\frac{1}{2}$   
 $-x^2 - \frac{1}{2}a + 3 = 0 \quad \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{\frac{1}{2} \pm \sqrt{\frac{1}{4} + 12}}{-2} \rightarrow -\frac{1}{2} = -2$   
 $\rightarrow +\frac{4}{2} = \frac{3}{2} = b$   
 $a+b = \frac{3}{2} - \frac{1}{2} = 1$

$\textcircled{1} x \geq 1 \rightarrow \sqrt{3x-2-x} = \sqrt{2x-2} \rightarrow 2x-2 \geq 0 \quad \boxed{x \geq 1}$

$\textcircled{2} x < 1 \rightarrow \sqrt{3x+2-x} = \sqrt{x+2} \rightarrow x+2 \geq 0 \quad x \geq -2 \rightarrow \boxed{-2 \leq x < 1}$

$\textcircled{1} \cup \textcircled{2} \rightarrow [-2, +\infty)$

$$f(x) = \sqrt{x} - f(x)$$

$$f\left(\sin \frac{\pi}{p} + \cos \sqrt{x}\right) = f\left(\frac{\sqrt{x}}{y} + 1\right)$$

$$\sqrt{\left(\frac{\sqrt{x}}{y} + 1\right)^2} - \sqrt{\left(\frac{\sqrt{x}}{y} + 1\right)} = \sqrt{\left(\frac{x}{y} + 1 + \sqrt{x}\right)} - \sqrt{\frac{x}{y}} - \sqrt{1} = \frac{x}{y} + 1 + \sqrt{x} - \sqrt{x} - 1 = \frac{x}{y}$$

✓  $\frac{1}{y}$

$$f(a) = \frac{a+1}{a-1}$$

$$f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{\frac{a+1+a-1}{a-1}}{\frac{a+1-a+1}{a-1}} = \frac{\frac{2a}{a-1}}{\frac{2}{a-1}} = \frac{2a}{2} = a$$

$$\frac{a(a-1)}{a-1} = a$$

$$a \neq 1 \Rightarrow D_f = \mathbb{R} - \{1\}$$

$$g(a) = \sqrt{a^2 - 1}$$

$$f(g(a)) = (\sqrt{a^2 - 1})^2 - \sqrt{a^2 - 1} = -\sqrt{a^2 - 1}$$

$$y^2 - \sqrt{y} + 1 = 0 \quad (y = \sqrt{a^2 - 1})^2 = 0 \rightarrow \sqrt{a^2 - 1} - \sqrt{y} = 0 \quad \sqrt{a^2 - 1} = \sqrt{y}$$

$$a^2 - 1 = y \quad a^2 = y + 1 \quad a = \pm \sqrt{y+1}$$

$$f(f) = 1 - \omega + y = x$$

$$x + y = 1 \quad (x=1)$$

$$f(-x) = x^2 + x + y = x^2 + x + 1 - x = x^2 + 1$$

$$x + y = -x \quad x = -y$$

$$\left. \begin{array}{l} f(f) = 1 - \omega + y = x \\ x + y = 1 \end{array} \right\} \quad \sqrt{x} = 1 - \sqrt{x}$$

$$\sqrt{\cos^2 x} + \omega \cos x = \sqrt{1 - \omega^2}$$

$$\sqrt{\cos^2 x} + \omega \cos x - \sqrt{1 - \omega^2} = 0 \quad \cos^2 x + \omega \cos x - 1 = 0$$

$$(\cos x + 1)(\cos x - 1) = 0$$

$$\cos x = \frac{1}{\sqrt{2}} \times \sqrt{2} = 1$$

$$\cos x = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \tan x = \sqrt{2} \rightarrow \tan^2 x = (\sqrt{2})^2 = 2$$

$$\Rightarrow f(x, \omega) = \sqrt{2}$$