

$$f\left(\sin \frac{\pi}{\sqrt{2}} + \cos \frac{\pi}{\sqrt{2}}\right) \quad f\left(\frac{1+\sqrt{2}}{\sqrt{2}}\right) = \sqrt{2} \left(\frac{1+\sqrt{2}}{\sqrt{2}}\right) - \sqrt{2} \left(\frac{1+\sqrt{2}}{\sqrt{2}}\right) = \frac{1+\sqrt{2}}{\sqrt{2}} - \frac{1+\sqrt{2}}{1} = \frac{1+\sqrt{2}}{\sqrt{2}} - \frac{1+\sqrt{2}}{\sqrt{2}} = -\frac{1}{\sqrt{2}}$$

$\downarrow \quad \downarrow \quad \rightarrow \frac{1+\sqrt{2}}{\sqrt{2}}$

6

$$f\left(\frac{a+1}{a-1}\right) = a \rightarrow f\left(\frac{a+1}{a-1}\right) = \frac{\frac{a+1}{a-1} + a - 1}{\frac{a+1}{a-1} - a + 1} = \frac{\frac{a+1+a(a-1)}{a-1}}{\frac{a+1-a(a-1)}{a-1}} = \frac{a+1+a(a-1)}{a+1-a(a-1)} = a = a \rightarrow \mathbb{R}$$

- $\boxed{a \neq 1} \rightarrow \frac{a+1}{a-1} \neq 1 \rightarrow a+1 \neq a-1 \leftarrow \mathbb{R} \rightarrow D_f = \mathbb{R} - \{1\}$

7

$$g(a) = \sqrt{a^2 - 1} \rightarrow f(\sqrt{a^2 - 1}) = (\sqrt{a^2 - 1})^2 - \sqrt{a^2 - 1} = |a^2 - 1| - \sqrt{a^2 - 1} = -\sqrt{a^2 - 1}$$

$$\begin{aligned} t^2 - \sqrt{a^2 - 1} t + \sqrt{a^2 - 1} &= 0 \\ (t - \sqrt{a^2 - 1})(t - 1) &= 0 \quad t = \sqrt{a^2 - 1} \\ \sqrt{a^2 - 1} &= \sqrt{a^2 - 1} \\ |a^2 - 1| &= \sqrt{a^2 - 1} \\ a^2 &= \delta \rightarrow a = \pm \sqrt{\delta} \end{aligned}$$

8

$$\begin{aligned} f(-3) &\rightarrow x + y = -3 \rightarrow x = -4 \rightarrow 3^4 + 3^0 + 4 = 81 \\ f(4) &\rightarrow x + y = 4 \rightarrow x = 1 \rightarrow 1 - 4 + 4 = 1 \end{aligned}$$

$\rightarrow 2 \times 81 = 162$

9

$$\begin{aligned} \sqrt{2} \cos^2 x + 2 \cos x &= \frac{1}{\sqrt{2}} \rightarrow 1 \cos^2 x + 1 \cos x - \frac{1}{\sqrt{2}} = 0 \rightarrow 1t^2 + 1t - \frac{1}{\sqrt{2}} = 0 \\ t = \cos x &= \frac{1}{\sqrt{2}} \rightarrow 45^\circ, 135^\circ \\ t = \cos x &= -\frac{1}{\sqrt{2}} \rightarrow 225^\circ, 315^\circ \rightarrow -1 \leq \cos x \leq 1 \\ 1 + \tan^2 x \cdot \frac{1}{\cos^2 x} &\rightarrow 1 + \tan^2 x \cdot \frac{1}{(\frac{1}{\sqrt{2}})^2} = 4 \Rightarrow \tan^2 x = 3 \\ &\rightarrow f(3, \delta) = 3 \end{aligned}$$

$$\begin{aligned} \Delta &= 1 - 4 \left(1 \cdot \left(-\frac{1}{\sqrt{2}}\right)\right) = 1 - 4 \left(-\frac{1}{\sqrt{2}}\right) = 1 + \frac{4}{\sqrt{2}} = \frac{\sqrt{2} + 4}{\sqrt{2}} \\ t &= \frac{-1 \pm \sqrt{\frac{\sqrt{2} + 4}{\sqrt{2}}}}{1} \rightarrow \frac{-1 + 1}{1} = 0 \\ &\rightarrow \frac{-1 - 1}{1} = -2 \end{aligned}$$

10

$$\text{مثال 1: } f(x) = \sqrt{\frac{x-1}{x}} - \frac{x}{x-1} = \sqrt{\frac{-x^2 + x^2 - x + 1}{x^2 - x}} = \sqrt{\frac{1-x}{x(x-1)}} \cdot \frac{1}{\frac{1}{x} - \frac{1}{x-1}} \rightarrow D_f = (-\infty, 0) \cup \left[\frac{1}{x}, 1\right)$$

$$\text{مثال 2: } f(x) = \frac{x - \sqrt{x^2 - 1}}{x(x+1)} = \frac{x - \sqrt{x^2 - 1}}{(x-1)(x+1)} \rightarrow x \neq 0, x \neq -1, x \neq 1, x \neq -\sqrt{x^2 - 1} \rightarrow D_f = \mathbb{R} - \left\{-\sqrt{x^2 - 1}, -\frac{1}{x}, -1, 0, 1\right\}$$

$$\text{مثال 3: } f(x) = \sqrt{\left(\frac{1}{x}\right)^2 - 9} \sqrt{x^2 - 4} = \sqrt{\frac{1 - 9x^2}{x^2}} \sqrt{x^2 - 4} \rightarrow D_f = (-\infty, -2] \cup \left[\frac{1}{9}, +\infty\right)$$

$$\text{مثال 4: } \sqrt{x-1} + \sqrt{y+1} = 0 \rightarrow \sqrt{x-1} = -\sqrt{y+1} \rightarrow x-1 \geq 0 \rightarrow x \geq 1 \rightarrow D_f = [1, \infty)$$

$$f(x) = \sqrt{x^2 - 1} - x - 1 \geq 0 \rightarrow \sqrt{x^2 - 1} \geq x + 1 \rightarrow x^2 - 1 \geq (x+1)^2 \rightarrow x^2 - 1 \geq x^2 + 2x + 1 \rightarrow -2x - 2 \geq 0 \rightarrow x \leq -1 \rightarrow D_f = (-\infty, -1] \cup (1, +\infty)$$

$$-x^2 + ax + 3 = 0 \rightarrow -x^2 - 1 = 0 \rightarrow x^2 = -1 \rightarrow x = \pm i \rightarrow a = \frac{1}{x} \rightarrow a = \pm i$$

$$f(x) = x \rightarrow f(x) \geq x \rightarrow x + 3 \geq x \rightarrow x \geq -3 \rightarrow D_f = [-3, +\infty)$$