

۲. آنزین های نوشتنی

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}}$ $\rightarrow \frac{(x-1)^2 - x^2}{(x-1)x} \geq 0 \rightarrow \frac{(x-1-x)(x-1+x)}{(x-1)x} \geq 0$ $\frac{0}{\frac{1}{x} - \frac{1}{x-1} + \frac{1}{x-1}}$
 $x \neq 0$ $x \neq 1$ $D_f = (-\infty, 0) \cup (\frac{1}{2}, 1)$

ب) $f(x) = \frac{\frac{1}{x+1} - \frac{2}{x}}{\frac{3}{x-1} + \frac{1}{x+2}}$ $\rightarrow \left[\frac{-x-2}{x(x+1)} \right] \cdot \frac{(-x-2)(x-1)(x+2)}{x(x+1)(x+2)}$ $D_f = \mathbb{R} - \{-2, -\frac{5}{2}, -1, 0, 1\}$

الف) $f(x) = \sqrt{\left(\left(\frac{1}{3}\right)^x - 9\right)(32 - 4^x)}$ $\rightarrow \sqrt{(3^{-x} - 9)(32 - 2^{2x})}$ $\frac{-2}{\frac{1}{3} - \frac{1}{3} + \frac{1}{3}}$ $(-\infty, -2) \cup (\frac{5}{2}, +\infty)$

ب) $\sqrt[4]{x-1} + \sqrt{y+1} = 3$ $\sqrt[4]{y+1} = 3 - \sqrt[4]{x-1}$ $\rightarrow 3 - \sqrt[4]{x-1} \geq 0 \rightarrow 3 \geq \sqrt[4]{x-1} \rightarrow 81 \geq x-1 \rightarrow 82 \geq x$ $D_f = [1, 82]$

$f(x) = \frac{\log_f(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$ $x^2 - x - 2 > 0 \rightarrow (x-2)(x+1)$ $\frac{-1}{\frac{1}{3} - \frac{1}{3} + \frac{1}{3}}$ $\cap$

$\rightarrow x^2 - 1 > 0 \rightarrow x^2 \geq 1 \rightarrow x \leq -1$ $x \geq 1$ $D_f = (-\infty, -1) \cup (2, +\infty)$

$f(x) = \sqrt{3 + ax - x^2}$ $D_f = [2, b]$

$x \rightarrow -2 \rightarrow 3 - 2a - 4 = 0 \rightarrow -1 = 2a \rightarrow a = -\frac{1}{2}$

$a+b = -\frac{1}{2} + \frac{4}{2} = 1$

$\frac{3}{a} = \frac{4}{-1} = -4$ $-4 + b \rightarrow b = \frac{4}{2}$

$f(x) = \begin{cases} 3x - 2 & x \geq 1 \\ 2x + 3 & x < 1 \end{cases}$

$g(x) = \sqrt{f(x) - x}$ $f(x) - x \geq 0$

$x \geq 1 \rightarrow 3x - 2 - x \geq 0 \rightarrow 2(x-1) \geq 0$ $x \geq 1$

$x < 1 \rightarrow 2x + 3 - x \geq 0 \rightarrow x + 3 \geq 0$ $x \geq -3$ $D_f = [-3, 1)$

$f\left(\underbrace{\sin \frac{\pi}{r}}_{\frac{\sqrt{r}}{r}} + \underbrace{\cos r\pi}_1\right)$ $f(x) = rx^r - rx \rightarrow r\lambda(\lambda - r)$ $\frac{\sqrt{r}}{r} + 1 \rightarrow \frac{r + \sqrt{r}}{r}$ $r + \sqrt{r} \left(\frac{r + \sqrt{r} - r}{r} \right)$ $r + \sqrt{r} \left(-\frac{r + \sqrt{r}}{r} \right) = \frac{r - r}{r} = -\frac{1}{r}$	6
$f(x) = \frac{x+1}{x-1} \rightarrow a \neq 1$ $f(a) = \frac{a+1}{a-1} \rightarrow f(f(a)) \rightarrow \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} \rightarrow \frac{\frac{r_a}{a-1}}{\frac{r}{a-1}} = a$ $\mathbb{R} - \{a \neq 1\}$	7
$g(x) = \sqrt{x^r - 1}, f(x) = x^r - rx$ $f(g(a)) = -r$ $g(a) = \sqrt{a^r - 1} = b$ $f(g(a)) = a^r - 1 - r(\sqrt{a^r - 1}) = -r \rightarrow a^r - 1 - r\sqrt{a^r - 1} + r = 0$ $b^r - rb + r = 0 \rightarrow (b-r)^r = 0$ $\begin{aligned} \sqrt{a^r - 1} - r &= 0 \\ \sqrt{a^r - 1} &= r \\ a^r - 1 &= r^2 \\ a^r &= r^2 + 1 \\ a &= \sqrt[r]{r^2 + 1} \end{aligned}$	8
$f(x+r) = x^r - ax + c$ $r + r = c \rightarrow r = 1 \rightarrow 1 - a + c = r$ $r + r = c \rightarrow r = -c \rightarrow r^2 + r = c + r^2$ $\underbrace{f(r)}_r, \underbrace{f(-r)}_{-r} = 1 \neq c$	9
$f(r \cos^r x + \Delta \cos x) = \tan^r x$ $f(r, a) =$ $\left(r \cos^r x + \Delta \cos x = \frac{r}{r}, \Delta = \frac{r}{r} \right) x^r \rightarrow r \cos^r x + \underbrace{(\Delta \cos x - \frac{r}{r})}_{p} = 0$ $r p^r + 1 \cdot p - r = 0$ $\Delta = 1 \rightarrow f(r)(-r) = r^2 r$ $\frac{-1 \pm 1r}{1r} \rightarrow \frac{1}{r} \rightarrow \left(\frac{r}{1} \right)^r \rightarrow (\sqrt{r})^r = r$ $-1 \leq \cos x \leq 1$	10