

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}}$ $\rightarrow \frac{(x-1)^2 - x^2}{(x-1)x} \geq 0 \rightarrow \frac{(x-1-x)(x-1+x)}{(x-1)x} \geq 0$ $D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $f(x) = \frac{\frac{1}{x+1} - \frac{2}{x}}{\frac{3}{x-1} + \frac{1}{x+2}}$ $\rightarrow \left[\frac{-x-2}{x(x+1)} \right] \cdot \frac{(-x-2)(x-1)(x+2)}{x(x+1)(x+2)}$ $D_f = \mathbb{R} - \left\{ -2, -\frac{5}{2}, -1, 0, 1 \right\}$

الف) $f(x) = \sqrt{\left(\left(\frac{1}{3}\right)^x - 9\right)(32 - 4^x)}$ $\rightarrow \sqrt{(3^{-x} - 9)(32 - 2^{2x})}$ $D_f = (-\infty, -2] \cup [\frac{5}{2}, +\infty)$

ب) $\sqrt[4]{x-1} + \sqrt[4]{y+1} = 3$ $\rightarrow \sqrt[4]{y+1} = 3 - \sqrt[4]{x-1} \geq 0 \rightarrow 3 \geq \sqrt[4]{x-1} \rightarrow 11 \geq x-1 \rightarrow 12 \geq x$
 $x-1 \geq 0 \rightarrow x \geq 1$ $D_f = [1, 12]$

$f(x) = \frac{\log_f(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$ $x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0$ $D_f = (-\infty, -1) \cup (2, +\infty)$

$\rightarrow x^2 - 1 > 0 \rightarrow x^2 \geq 1 \rightarrow x \leq -1 \quad x \geq 1$
 $\sqrt{x^2 - 1} + 1 \neq 0$
 $\sqrt{x^2 - 1} \neq -1$

$f(x) = \sqrt{3 + ax - x^2}$ $D_f = [2, b]$

$x \rightarrow -2 \rightarrow 3 - 2a - 4 = 0 \rightarrow -1 = 2a \rightarrow a = -\frac{1}{2}$

$a \cdot b = -\frac{1}{2} + \frac{3}{2} = 1$

$\frac{3}{a} = \frac{4}{-1} = -4 \rightarrow -4 \times b \rightarrow b = \frac{4}{-4}$

$f(x) = \begin{cases} 3x - 2 & x \geq 1 \\ 2x + 3 & x < 1 \end{cases}$

$g(x) = \sqrt{f(x) - x}$ $\rightarrow x \geq 1 \rightarrow 3x - 2 - x \geq 0 \rightarrow 2(x-1) \geq 0$
 $x < 1 \rightarrow 2x + 3 - x \geq 0 \rightarrow x + 3 \geq 0 \rightarrow x \geq -3$ $D_f = [-3, +\infty)$

$f\left(\underbrace{\sin \frac{\pi}{r}}_{\frac{\sqrt{r}}{r}} + \underbrace{\cos r\pi}_1\right)$ $f(x) = rx^r - rx \rightarrow r\lambda(\lambda - r)$ $\frac{r + \sqrt{r}}{r} \rightarrow \frac{r + \sqrt{r}}{r}$ $r + \sqrt{r} \left(\frac{r + \sqrt{r} - r}{r} \right)$ $r + \sqrt{r} \left(-\frac{r + \sqrt{r}}{r} \right) = \frac{r - r}{r} = \boxed{-\frac{1}{r}}$	6
$f(x) = \frac{x+1}{x-1} \rightarrow a \neq 1$ $f(a) = \frac{a+1}{a-1} \rightarrow f(f(a)) \rightarrow \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} \rightarrow \frac{\frac{r_a}{a-1}}{\frac{r}{a-1}} = a$ $\mathbb{R} - \{a \neq 1\}$	7
$g(x) = \sqrt{x^r - 1}, f(x) = x^r - rx$ $f(g(a)) = -r$ $g(a) = \sqrt{a^r - 1} = b$ $f(g(a)) = a^r - 1 - r(\sqrt{a^r - 1}) = -r \rightarrow a^r - 1 - r\sqrt{a^r - 1} + r = 0$ $b^r - rb + r = 0 \rightarrow (b-r)^r = 0$ $\sqrt{a^r - 1} - r = 0$ $\sqrt{a^r - 1} = r$ $a^r - 1 = r^2$ $a^r = 1 + r^2$ $a = \sqrt[1]{1 + r^2}$	8
$f(x+r) = x^r - rx + r$ $r + r = r \rightarrow r = 1 \rightarrow 1 - a + r = r$ $r + r - r \rightarrow r = -r \rightarrow r^2 + r^2 + r = \sqrt{r}$ $\underbrace{f(r)}_r, \underbrace{f(-r)}_{\sqrt{r}} = 1 \neq$	9
$f(r \cos^r x + \Delta \cos x) = \tan^r x$ $f(p, a) =$ $\left(r \cos^r x + \Delta \cos x = p, \Delta = \frac{p}{r} \right) x^r \rightarrow r \cos^r x + \underbrace{10 \cos x - \sqrt{2}}_p = 0$ $r p^r + 10 p - \sqrt{2} = 0$ $\Delta = 10 - r(\lambda)(-\sqrt{2}) = 344r$ $\frac{-10 \pm 18}{14} \rightarrow \boxed{\frac{1}{r}} \rightarrow \left(\frac{\sqrt{p}}{\frac{1}{r}} \right)^r \rightarrow (\sqrt{p})^r = \boxed{p}$ $\frac{-10 \pm 18}{14} \rightarrow -1 \leq \cos x \leq 1$	10