

$$f(x) \begin{cases} (a+1)(x+1) & : x > 1 \\ 2a+2x & : x \leq 1 \end{cases}$$

$$\begin{aligned} \rightarrow 2f(a) &= f(1+a) \Rightarrow 2(a+1)(a+1) = 2a-2+a \\ &\Rightarrow 2a+2 = 2a-2+a \Rightarrow a = -4 \\ &\Rightarrow 2a+2 = -8+2 = -6 \end{aligned}$$

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$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2 = \frac{x+1}{\sqrt{x}} + 2$$

$$\frac{1}{\sqrt{x}} \rightarrow x = 2\sqrt{3} \Rightarrow \frac{1}{\sqrt{2\sqrt{3}}} = \frac{1}{\sqrt{2}\sqrt{\sqrt{3}}} = \frac{1}{\sqrt{2}\sqrt[4]{3}} = \frac{1}{\sqrt{2}\sqrt[4]{3}}$$

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$$\begin{aligned} &\rightarrow \sqrt{2+3} + \sqrt{2-3} + 2\sqrt{2+3} + \sqrt{2-3} + 2 \\ &= 2 + 2(\sqrt{2+3} + \sqrt{2-3}) = 2 + 2\sqrt{6} \\ &\rightarrow \sqrt{2+3} + \sqrt{2-3} + 2 = 6 \rightarrow \sqrt{6} \end{aligned}$$

$$2f(x) - 3f(-x) = 5x^2 + x$$

$$\rightarrow f(x) = ax^2 + bx + c \Rightarrow 2ax^2 + 2bx + 2c - 3(-ax^2 - 3bx - 3c) = 5x^2 + x$$

$$\Rightarrow -ax^2 + 6bx - c = 5x^2 + x \rightarrow c = 0, -a = 5 \Rightarrow a = -5, 6b = 1 \Rightarrow b = \frac{1}{6}$$

$$\Rightarrow f(x) = -5x^2 - \frac{1}{6}x$$

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$$(x+1)f(a) - 2xf(x+1) = 5x^2 - mx + m-1$$

$$x = -1 \Rightarrow 9f(a) = 19 + 2m + m-1$$

$$x = a \Rightarrow 2f(a) = 2m-1 \Rightarrow 9f(a) = 9m-3 \Rightarrow 18+2m = 9m-3 \Rightarrow 11 = 7m \Rightarrow m = \frac{11}{7}$$

$$9f(a) = 9 \times \frac{9}{7} - 3 = \frac{81}{7} - \frac{21}{7} = \frac{60}{7}$$

$$\Rightarrow f(a) = \frac{60}{7 \times 9} = \frac{10}{7}$$

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$$f(x) + f\left(\frac{1}{x}\right) = \frac{2x^2 - 12x + 3}{x} \rightarrow x = -1 \Rightarrow f(-1) + f\left(\frac{1}{-1}\right) = \frac{2+12+3}{-1} = -17$$

$$\Rightarrow f(-1) = -9$$

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$$f(x) = (x-1)\sqrt{1-x} \rightarrow (1-x) \geq 0 \Rightarrow D_f = (-\infty, 1] \rightarrow D_f = D_g$$

$$g(x) = -\sqrt{(1-x)^3} \rightarrow (1-x) \geq 0 \Rightarrow D_g = (-\infty, 1]$$

$$(x-1)\sqrt{1-x} = -(1-x)\sqrt{1-x} = -\sqrt{(1-x)^2(1-x)} = -\sqrt{(1-x)^3}$$

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$$f(x) = x\sqrt{x} \rightarrow x \geq 0 \rightarrow D_f = [0, +\infty) \rightarrow D_f = D_g \mid x\sqrt{x} = \sqrt{x^3} \Rightarrow \text{دو تابع برابرند}$$

$$g(x) = \sqrt{x^3} \rightarrow x \geq 0 \rightarrow D_g = [0, +\infty)$$

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$$f(x) = [\sqrt{x} - \lfloor \sqrt{x} \rfloor] \quad x \geq 0 \rightarrow D_f = [0, +\infty) \rightarrow D_f = D_g \mid x \in [0, +\infty) \Rightarrow x - \lfloor x \rfloor = 0$$

$$g(x) = \sqrt{x} - \lfloor x \rfloor \rightarrow x \geq 0 \rightarrow D_g = [0, +\infty)$$

$$\sqrt{x} \in \mathbb{N} \Rightarrow \sqrt{x} - \lfloor \sqrt{x} \rfloor = 0$$

$$\sqrt{x} \notin \mathbb{N} \Rightarrow 0 < \sqrt{x} - \lfloor \sqrt{x} \rfloor < 1 \Rightarrow [\sqrt{x} - \lfloor \sqrt{x} \rfloor] = 0 \Rightarrow f(x) = 0 = g(x) \Rightarrow \text{دو تابع مساوی اند}$$

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$$g = \{(-1, 1), (1, 1), (2, 3), (3, 5), (0, 2), (\sqrt{2}, 1)\} \quad f = \{(1, 2), (2, 3), (3, 1), (0, 0)\}$$

$$\text{الف) } \left[\frac{f+g}{f-g} \right] (1) = \frac{2+1}{2-1} = 3 \quad \text{ب) } \left[\frac{f+g}{f-g} \right] (2) = \frac{3+3}{3-3} \rightarrow \text{تخریف نشده}$$

$$\text{ج) } f(0) \cdot g(1) \neq 0 \times 1 = 0$$

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$$f(x) = \{(2, 1), (-1, 4), (0, 5), (-2, 2), (3, -1)\}$$

$$\text{الف) } g(x) = \{x, x-1\} \Rightarrow g(x) = \{(5, 1), (4, 4), (3, 5), (2, 2), (1, -1)\}$$

$$g(x) = f(3x-1) = \{(1, 1), (0, 4), \left(\frac{1}{3}, 5\right), \left(-\frac{1}{3}, 2\right), \left(\frac{4}{3}, -1\right)\}$$

$$h(x) = f(1+x) = \{(2, 1), (-2, 1), (0, 5), (3, -1), (-3, -1)\}$$

$$\cancel{1+x} = -x$$

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