

$$f(n) = \begin{cases} (a+1)(n+2) & n \geq 1 \\ 3a + 2n & n \leq 1 \end{cases}$$

$$2f(5) = f(-2) + a$$

$$2(a+1)(5+2) = 3a + 2(-2) + a$$

$$14a + 14 = 4a - 4$$

$$10a = -18 \Rightarrow a = -1.8$$

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$$f(n) = \sqrt{n} + \frac{1}{\sqrt{n}} + 2$$

$$f(1-\sqrt{3}) + f(1+\sqrt{3}) = ?$$

$$\sqrt{1-\sqrt{3}} + \frac{1}{\sqrt{1-\sqrt{3}}} + 2 + \sqrt{1+\sqrt{3}} + \frac{1}{\sqrt{1+\sqrt{3}}} + 2 \rightarrow \frac{\sqrt{1-\sqrt{3}} + \sqrt{1+\sqrt{3}}}{\sqrt{1-\sqrt{3}}\sqrt{1+\sqrt{3}}} + \frac{1}{\sqrt{1-\sqrt{3}}} + \frac{1}{\sqrt{1+\sqrt{3}}} + 4$$

$$\frac{\sqrt{1-\sqrt{3}} + \sqrt{1+\sqrt{3}}}{\sqrt{1-3}} + \frac{\sqrt{1+\sqrt{3}} + \sqrt{1-\sqrt{3}}}{\sqrt{1-3}} + 4$$

$$\frac{\sqrt{1-\sqrt{3}} + \sqrt{1+\sqrt{3}}}{\sqrt{-2}} + \frac{\sqrt{1+\sqrt{3}} + \sqrt{1-\sqrt{3}}}{\sqrt{-2}} + 4$$

$$2\sqrt{1-\sqrt{3}} + 2\sqrt{1+\sqrt{3}} + 4 = A$$

$$2(\sqrt{1-\sqrt{3}} + \sqrt{1+\sqrt{3}}) = A - 4 \xrightarrow{\text{مربع کن}} 4(1-\sqrt{3} + 2\sqrt{1-\sqrt{3}}\sqrt{1+\sqrt{3}} + 1+\sqrt{3}) = (A-4)^2$$

$$4(2 + 2\sqrt{1-\sqrt{3}}\sqrt{1+\sqrt{3}}) = (A-4)^2$$

$$8 + 8\sqrt{1-\sqrt{3}}\sqrt{1+\sqrt{3}} = (A-4)^2$$

$$8 + 8\sqrt{1-3} = (A-4)^2$$

$$8 - 8 = (A-4)^2$$

$$0 = (A-4)^2$$

$$A-4 = 0$$

$$A = 4$$

$$f(1-\sqrt{3}) + f(1+\sqrt{3}) = 4$$

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$$2f(n) - 3f(-n) = 4n^2 - n \rightarrow \text{برای } n \text{ صحیح و صحیح کسری}$$

$$2f(-n) - 3f(n) = 4n^2 + n$$

$$\begin{cases} (2f(n) - 3f(-n) = 4n^2 - n) \times 2 \\ (2f(-n) - 3f(n) = 4n^2 + n) \times 3 \end{cases} \rightarrow \begin{cases} 4f(n) - 6f(-n) = 8n^2 - 2n \\ 6f(-n) - 9f(n) = 12n^2 + 3n \end{cases}$$

$$-5f(n) = 4n^2 - n$$

$$f(n) = -\frac{4n^2 - n}{5}$$

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$$(n+2)f(n) - 3nf(n+2) = 4n^2 + 2n - 1$$

$$n=0 \rightarrow 2f(0) = 3m - 1 \rightarrow f(0) = \frac{3m-1}{2}$$

$$n=-2 \rightarrow 4f(0) = 4(-2)^2 - m(-2) + 3m - 1$$

$$4f(0) = 16 + 2m$$

$$f(0) = \frac{16+2m}{4}$$

$$\frac{16+2m}{4} = \frac{3m-1}{2}$$

$$16+2m = 2(3m-1)$$

$$16+2m = 6m-2$$

$$m = 4$$

$$f(0) = \frac{3m-1}{2} \rightarrow f(0) = \frac{3(4)-1}{2} = \frac{11}{2}$$

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$$f(n) + f\left(\frac{1}{n}\right) = \frac{3n^2 - 12n + 4}{n}$$

$$f(n) = an + b$$

$$an + b + \frac{a}{n} + b = \frac{3n^2 - 12n + 4}{n} \rightarrow \text{از سمت راست، ضرایب } n^2, n, \text{ و ثابت را جدا می‌کنیم}$$

$$\frac{an^2 + a + bn + b}{n} = \frac{3n^2 - 12n + 4}{n} \Rightarrow \begin{cases} a = 3 \\ b = -12 \\ b = 4 \end{cases}$$

$$f(n) = 3n - 4$$

$$f(-1) + f(-\frac{1}{2}) = 4$$

$$f(-1) = -7$$

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$\begin{cases} f(n) = (n-1)\sqrt{1-n} \rightarrow Df = \frac{1-n}{\sqrt{1-n}} \\ g(n) = -\sqrt{(1-n)^3} \rightarrow Dg = (1-n)^{\frac{3}{2}} \rightarrow g(n) = - 1-n \sqrt{1-n} \end{cases}$ این دو تابع در $1-n=1-n$ و $1-n=1-n$ یکسانند در نتیجه $f(n) = g(n)$ $Df = Dg$ $x = -5$ $f(-5) = -6\sqrt{6}$ $g(-5) = -\sqrt{(-6)^3} = -6\sqrt{6}$	۶
$\begin{aligned} f(n) &= n\sqrt{n} & Df &= \frac{3}{2}n^{\frac{1}{2}} \\ g(n) &= \sqrt{n^3} & Dg &= \frac{3}{2}n^{\frac{1}{2}} \rightarrow \frac{3}{2}n^{\frac{1}{2}} \end{aligned}$ $Df = Dg$ ✓ $g(n) = n \sqrt{n} \rightarrow$ چون $n = n$ پس $g(n) = n\sqrt{n} = f(n)$	۷
$\begin{aligned} f(n) &= [\sqrt{n} - \sqrt{n-1}] & Df &= \frac{1}{2\sqrt{n}} - \frac{1}{2\sqrt{n-1}} \\ g(n) &= \sqrt{n-1} & Dg &= \frac{1}{2\sqrt{n-1}} \end{aligned}$ $Df = Dg$ $f(n) = [\sqrt{n} - \sqrt{n-1}] \rightarrow 0 \rightarrow$ در این نقطه $f(n) = 0$ و $g(n) = 0$ پس $f(n) = g(n)$	۸
$f = \{(1,2), (2,3), (3,4), (4,5)\} \quad g = \{(1,1), (2,2), (3,3), (4,4)\}$ الف) $\left(\frac{f+g}{f-g}\right)(1) = \frac{1+1}{1-1} = 2$ ب) $\left(\frac{f+g}{f-g}\right)(2) = \frac{2+2}{2-2} = \frac{4}{0}$ (تقسیم بر صفر) ج) $f(0) \cdot g(-1) = 0 \times 1 = 0$	۹
$f(n) = \{(1,1), (-1,1), (0,0), (-2,2), (3,-1)\}$ الف) $g(n) = f(n-1)$ $n-1=1 \rightarrow n=2$ $n=1$ $n-1=-1 \rightarrow n=0$ $n=0$ $n-1=0 \rightarrow n=1$ $n=1$ $n-1=-2 \rightarrow n=-1$ $n=-1$ $n-1=3 \rightarrow n=4$ $n=4$ $g(n) = \{(0,1), (-2,1), (-1,0), (-3,2), (2,-1)\}$	۱۰