

پایه آفرین خوب نوستر

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$$f(x) = \begin{cases} (x+1)(x+2) & ; x > 1 \\ 2x + 2x & ; x \leq 1 \end{cases}$$

$$a = x \rightarrow (x+1)(x+2) = 2x - 1 + a$$

$$x = -2 \rightarrow 2a - 1$$

$$f(a) = f(-2) + a$$

$$a = ?$$

$$14a + 14 = 2a - 1$$

$$10a = -15$$

$$a = -1.5$$

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$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2$$

$$f(\sqrt{2}-\sqrt{3}) \rightarrow \sqrt{\sqrt{2}-\sqrt{3}} + \frac{1}{\sqrt{\sqrt{2}-\sqrt{3}}} + 2$$

$$f(\sqrt{2}-\sqrt{3}) + f(\sqrt{2}+\sqrt{3}) =$$

$$f(\sqrt{2}+\sqrt{3}) \rightarrow \sqrt{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{\sqrt{2}+\sqrt{3}}} + 2$$

$$\sqrt{\sqrt{2}+\sqrt{3}} + \sqrt{\sqrt{2}-\sqrt{3}} + \frac{1}{\sqrt{\sqrt{2}+\sqrt{3}}} + \frac{1}{\sqrt{\sqrt{2}-\sqrt{3}}} + 4$$

$$\sqrt{(\sqrt{2}+\sqrt{3}) + (\sqrt{2}-\sqrt{3})} + \sqrt{(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})} + 4$$

$$\sqrt{2\sqrt{2}} + \sqrt{2-3} + 4 = \sqrt{2\sqrt{2}} - 1 + 4 = \sqrt{2\sqrt{2}} + 3$$

$$\frac{1}{\sqrt{2}-\sqrt{3}} \times \frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}+\sqrt{3}} = \frac{\sqrt{2}+\sqrt{3}}{2-3} = \frac{\sqrt{2}+\sqrt{3}}{-1} = -\sqrt{2}-\sqrt{3}$$

$$\frac{1}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}} = \frac{\sqrt{2}-\sqrt{3}}{2-3} = \frac{\sqrt{2}-\sqrt{3}}{-1} = -\sqrt{2}+\sqrt{3}$$

$$\sqrt{2\sqrt{2}} + \sqrt{2-3} + 4 = \sqrt{2\sqrt{2}} - 1 + 4 = \sqrt{2\sqrt{2}} + 3$$

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$$\frac{d}{dx} \left(\frac{1}{x} f(x) - \frac{1}{x} f(-x) \right) = \frac{f(x) - f(-x)}{x^2} - \frac{f(x) + f(-x)}{x^2}$$

$$- \frac{1}{x^2} \rightarrow \frac{1}{x^2} f(x) - \frac{1}{x^2} f(-x) = \frac{f(x) - f(-x)}{x^2} + \frac{f(x) + f(-x)}{x^2}$$

$$- \frac{1}{x^2} \rightarrow \frac{1}{x^2} f(x) - \frac{1}{x^2} f(-x) = \frac{f(x) - f(-x)}{x^2} + \frac{f(x) + f(-x)}{x^2}$$

$$x \rightarrow \left(-\frac{d}{dx} f(x) = \frac{f(x)}{x^2} + \frac{1}{x^2} x \right) \rightarrow -\frac{d}{dx} f(x) = \frac{f(x)}{x^2} + \frac{1}{x}$$

$$f(x) = -\frac{f(x)}{x^2} - \frac{1}{x}$$

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$$\left(\frac{9}{x^2} + 1 \right) f(x) - \frac{1}{x} f(x+2) = \frac{f(x)}{x^2} - \frac{f(x+2)}{x} + \frac{f(x)}{x^2} - \frac{f(x+2)}{x}$$

$$f(0) = ?$$

$$x+2 = 0$$

$$x = -2$$

$$0 = x \rightarrow \frac{9}{x^2} f(x) = \frac{f(x)}{x^2} - 1 \rightarrow f(0) = \frac{f(x)}{x^2} - 1$$

$$-2 = x \rightarrow \frac{9}{x^2} f(x) = \frac{f(x)}{x^2} - 1 \rightarrow \frac{9}{4} f(-2) = \frac{f(-2)}{4} - 1$$

$$\frac{9}{4} f(-2) - \frac{f(-2)}{4} = -1$$

$$\frac{8}{4} f(-2) = -1$$

$$2 f(-2) = -1$$

$$f(-2) = -\frac{1}{2}$$

$$f(0) = \frac{1}{x^2} f(x) - 1 \Rightarrow \frac{1}{x^2} f(x) = f(0) + 1$$

$$f(x) = x^2 (f(0) + 1)$$

$$f(x) = x^2 \left(-\frac{1}{2} + 1 \right) = \frac{x^2}{2}$$

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$$f(x) + f\left(\frac{1}{x}\right) = \frac{2x^2 - 12x + 10}{x}$$

$$(ax + b + \frac{a}{x} + b) = \frac{2x^2 - 12x + 10}{x}$$

$$f(-1) = ?$$

$$ax^2 + bx + a + bx = 2x^2 - 12x + 10$$

$$f(x) = ax + b \rightarrow f\left(\frac{1}{x}\right) = \frac{a}{x} + b$$

$$ax^2 + bx + a + bx = 2x^2 - 12x + 10$$

$$a = 2 \quad 2b = -12 \rightarrow b = -6$$

$$f(x) = 2x - 6$$

$$f(-1) = 2(-1) - 6 = -8$$

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$\begin{cases} f(x) = (x-1)\sqrt{1-x} \geq 0 & D_f = x \leq 1 \\ g(x) = -\sqrt{(1-x)^3} \geq 0 & D_g = x \leq 1 \end{cases} \quad D_f = D_g$ سویلا تابع را بنویس $g(x) = -\sqrt{(1-x)^3} \rightarrow -\sqrt{(1-x)^2(1-x)} \rightarrow \frac{(x-1)}{(1-x)}\sqrt{1-x} = f(x)$ $\Rightarrow R_f = R_g$	۶
$\begin{cases} f(x) = x\sqrt{x} \geq 0 & x \geq 0 \\ g(x) = \sqrt{x^3} \geq 0 \rightarrow x \geq 0 \end{cases} \quad D_f = D_g$ سویلا تابع را بنویس $f(x) = x\sqrt{x} \rightarrow \sqrt{x \cdot x^2} = \sqrt{x^3} \Rightarrow R_f = R_g$	۷
$\begin{cases} f(x) = [\sqrt{x} - \lfloor \sqrt{x} \rfloor] \rightarrow x \geq 0 \\ g(x) = \sqrt{x - \lfloor x \rfloor} \quad x \geq 0 \end{cases} \quad D_f = D_g$ سویلا تابع را بنویس مثال: $R_f = R_g$	۸
$g = \{(-1, 1), (1, 1), (2, 3), (3, 2), (0, 2), (\sqrt{2}, 1)\} \quad f = \{(1, 2), (2, 3), (3, 1), (0, 0)\}$ ۱) $\left(\frac{f+g}{f-g}\right)(1) = \frac{1+1}{1-1} = \text{تعریف نشده}$ ۲) $\left(\frac{f+g}{f-g}\right)(2) = \frac{2+3}{2-3} = -5$ ۳) $f(0)g(-1) = 0$	۹
$f(x) = \{(2, 1), (-1, 4), (0, 2), (-2, 2), (3, -1)\}$ ۱) $g(x) = f(2x-1)$ $2x-1 = 2 \Rightarrow x = \frac{3}{2} \Rightarrow g(\frac{3}{2}) = f(2) = 1$ $2x-1 = -1 \Rightarrow x = 0 \Rightarrow g(0) = f(-1) = 4$ $2x-1 = 0 \Rightarrow x = \frac{1}{2} \Rightarrow g(\frac{1}{2}) = f(0) = 2$ $2x-1 = -2 \Rightarrow x = -\frac{1}{2} \Rightarrow g(-\frac{1}{2}) = f(-2) = 2$ $2x-1 = 3 \Rightarrow x = \frac{4}{2} = 2 \Rightarrow g(2) = f(3) = -1$ ۲) $h(x) = f(x)$ $x = 2 \Rightarrow h(2) = f(2) = 1$ $x = -1 \Rightarrow h(-1) = f(1) = 1$ $x = 3 \Rightarrow h(3) = f(3) = -1$ $x = -2 \Rightarrow h(-2) = f(2) = 1$ $h(x) = \{(-2, -1), (-1, 1), (0, 2), (1, 1), (2, -1)\}$	