

$$f(x) = \begin{cases} (a+1)(x+1) & ; x > 1 \\ 3a+2x & ; x \leq 1 \end{cases} \quad \text{و } f(a) = f(-1) + a$$

$$a > 1 \rightarrow f(a) = (a+1)(1) = a+1$$

$$-1 \leq a \rightarrow f(-1) = 3a+2(-1) = 3a-2$$

$$f(a) = f(-1) + a \rightarrow a+1 = 3a-2+a$$

$$1a-2a = -1 \rightarrow -1a = -1$$

$$1a = 1 \rightarrow a = 1$$

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2 \quad f(x-\sqrt{x}) + f(x+\sqrt{x}) = ?$$

$$(x-\sqrt{x})(x+\sqrt{x}) = 1$$

$$\text{و } t = \sqrt{x-\sqrt{x}}$$

$$\frac{1}{t} = \sqrt{x+\sqrt{x}} \quad \text{و } f(x+\sqrt{x}) = \frac{1}{t} + t + 2$$

$$f(x-\sqrt{x}) = t + \frac{1}{t} + 2$$

$$2 \left(t + \frac{1}{t} + 2 \right)$$

$$t + \frac{1}{t} = \sqrt{x-\sqrt{x}} + \sqrt{x+\sqrt{x}} \xrightarrow{\text{و } (x-\sqrt{x}) + (x+\sqrt{x})} 2x = 4$$

$$\text{و } t + \frac{1}{t} = \sqrt{2} \quad \text{و } f(x-\sqrt{x}) + f(x+\sqrt{x}) = 2(\sqrt{2} + \sqrt{2} + 2) = 4\sqrt{2} + 4$$

$$x < 0 \rightarrow f(-x) - 3f(x) = 2x^2 + x$$

$$\begin{cases} f(x) - 3f(-x) = 2x^2 - x \\ -3f(x) + f(-x) = 2x^2 + x \end{cases}$$

$$\begin{cases} 4f(x) - 4f(-x) = 4x^2 - 4x \\ -4f(x) + 4f(-x) = 4x^2 + 4x \end{cases}$$

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$$-4f(x) = 4x^2 + 4x$$

$$f(x) = -x^2 - x$$

$$n=3, \quad 4f(0) = 3m-1 \rightarrow f(0) = \frac{3m-1}{4}$$

$$n=4, \quad 7f(0) = 14 + 2m + 3m-1 \rightarrow 7f(0) = 12 + 5m$$

$$\text{از طرفی} \rightarrow 4f(0) = 3(3m-1) = 9m-3$$

$$\text{پس} \rightarrow 9m-3 = 12+5m$$

$$4m = 15$$

$$m = \frac{15}{4}$$

$$f(0) = \frac{3\left(\frac{15}{4}\right) - 1}{4}$$

$$= \frac{\frac{45}{4} - 1}{4} = \frac{\frac{41}{4}}{4} = \frac{41}{16} = f(0)$$

$$f(x) = ax + b$$

$$f(x) + f\left(\frac{1}{x}\right) = ax + b + \frac{a}{x} + b$$

$$f\left(\frac{1}{x}\right) = \frac{a}{x} + b$$

$$= ax + b + \frac{a}{x}$$

$$\frac{4x^4(4x+3)}{x} = 4x - 1 + \frac{3}{x}$$

$$4x + \frac{3}{x} - 1 = ax + b + \frac{a}{x}$$

$$\text{پس} \rightarrow a = 4$$

$$4b = -1 \rightarrow b = -\frac{1}{4}$$

$$f(x) = 4x - \frac{1}{4}$$

$$f(-1) = 4(-1) - \frac{1}{4} = -\frac{17}{4}$$

$$f(x) = (x-1)\sqrt{1-x}$$

$$g(x) = -\sqrt{(1-x)^3}$$

برای $f(x)$ و $g(x)$ هر دو
در بازه $[0, 1]$ تعریف شده اند

$$1-x \geq 0 \rightarrow x \leq 1$$

$$D_g = (-\infty, 1]$$

$$D_f = (-\infty, 1]$$

$$D_g = D_f$$

$$g(x) = -(1-x)\sqrt{1-x}$$

$$g(x) = (x-1)\sqrt{1-x}$$

$$g(x) = f(x)$$

$$\begin{cases} f(x) = x\sqrt{x} \rightarrow x > 0 \\ g(x) = \sqrt{x^3} \end{cases}$$

سین هر دو تابع هم دامنه
و هم ضابطه یکسان

$x > 0$

$$\sqrt{x^3} = x^{\frac{3}{2}}$$

$$\sqrt{x^3} = x^{\frac{3}{2}}$$

(مطابق)

$$Df = Dg$$

$$f(x) = g(x)$$

$$\begin{cases} f(x) = [\sqrt{x} - [\sqrt{x}]] \rightarrow x > 0 \\ g(x) = \sqrt{x - |x|} \end{cases}$$

$$[\sqrt{x}] \in \sqrt{x} \text{ یعنی } \sqrt{x}$$

$$Df = [0, +\infty)$$

$$Rf = \{0\} \text{ چون هر دو خروجی صفر است}$$

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$$|x| = x \text{ اگر } x \geq 0$$

$$g(x) = \sqrt{x - x} = 0 \text{ اگر } x \geq 0$$

$$|x| = -x \text{ اگر } x < 0$$

$$x - |x| = x - (-x) = 2x \text{ اگر } x < 0$$

$$2x < 0 \text{ چون } x < 0$$

بنابراین تعریف نشود

$$Dg = [0, +\infty)$$

$$Rg = \{0\}$$

(مطابق)

$$\left(\frac{f+g}{f-g} \right)(1) \rightarrow \frac{f(1)+g(1)}{f(1)-g(1)} = \frac{1+1}{1-1} = \frac{2}{0} = \infty$$

$$\Rightarrow \left(\frac{f+g}{f-g} \right)(x) \rightarrow \frac{f(x)+g(x)}{f(x)-g(x)} = \frac{x+x}{x-x} = \frac{2x}{0} = \infty$$

$$c) \frac{f(0)g(-1)}{g(-1)} = \frac{f(0)g(-1)}{1} = \frac{0 \times 1}{1} = 0$$

برای به دست آوردن $3x-1$ یکی از ورودی ها تابع f می باشد

$$\text{if } 3x-1 = 2 \rightarrow 3x = 3$$

$$x = 1$$

$$g(1) = f(2) = 1$$

$$\text{if } 3x-1 = -1 \rightarrow 3x = 0$$

$$x = 0$$

$$g(0) = f(-1) = 2$$

$$\text{if } 3x-1 = 0 \rightarrow 3x = 1$$

$$x = \frac{1}{3}$$

$$g\left(\frac{1}{3}\right) = f(0) = 0$$

$$\text{if } 3x-1 = -2 \rightarrow 3x = -1$$

$$x = -\frac{1}{3}$$

$$g\left(-\frac{1}{3}\right) = f(-2) = 2$$

$$\text{if } 3x-1 = 3 \rightarrow 3x = 4$$

$$x = \frac{4}{3}$$

$$g\left(\frac{4}{3}\right) = f(3) = -1$$

$$\xrightarrow{\text{مثال}} g = \left\{ (1, 1), (0, 2), \left(\frac{1}{3}, 0\right), \left(-\frac{1}{3}, 2\right), \left(\frac{4}{3}, -1\right) \right\}$$

$$\therefore D_f = \{-2, -1, 0, 2, 3\}$$

با توجه به این که f یک تابع گسسته است فقط این ها

می توانند باشند: $|x| = 0, 2, 3$

$$x = 0 : |x| = 0 \text{ برای}$$

$$h(0) = f(0) = 0$$

$$x = \pm 3 : |x| = 3 \text{ برای}$$

$$h(\pm 3) = f(3) = -1$$

$$x = \pm 2 : |x| = 2 \text{ برای}$$

$$h(\pm 2) = f(2) = 1$$

$$\xrightarrow{\text{مثال}} h = \{(-3, -1), (-2, 1),$$

$$(0, 0), (2, 1), (3, -1)\}$$